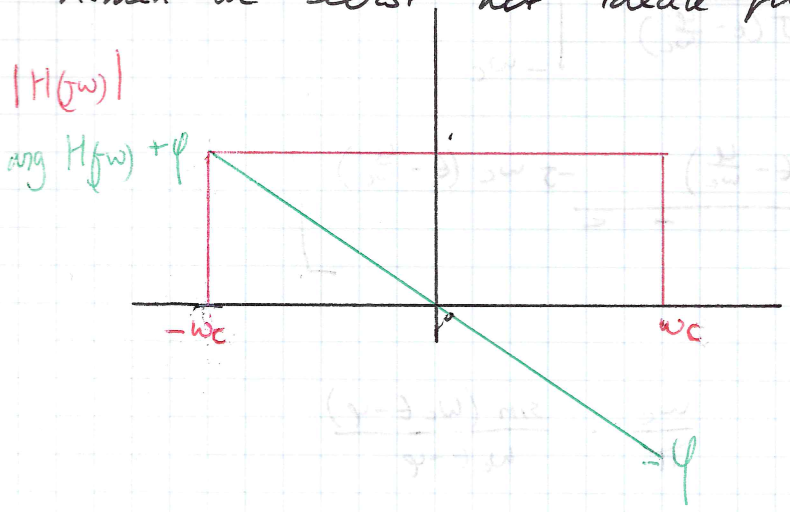
Filters

We gaan uit van L.D. filter :

Nemen we eerst het ideale filter



Is dit te realiseren ?
Bepalen we hiertoe de impuls responsie van het netwerk
$U_{in} = \delta(t)$

Bepaal eerst het spectrum :

$$\int_{-\infty}^{\infty} e^{-j\omega t}\,\delta(t)\,dt = 1$$

De impuls respontie in het f-domein =

$$1 \cdot H(j\omega) = H(j\omega)$$

In het t-domein :

$$\text{impulsresp} = \mathcal{F}^{-1}\{H(j\omega)\}$$

We berekenen deze :

$$\rightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t}\,H(j\omega)\,d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} e^{j\omega t}\cdot 1 \cdot e^{-j\frac{\varphi}{\omega_c}\cdot\omega}\,d\omega$$

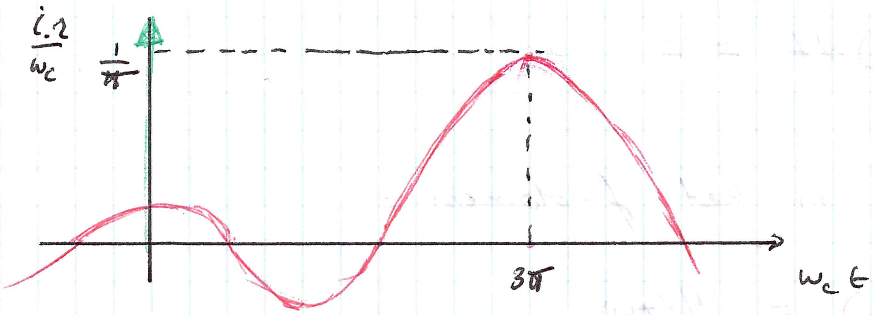
$$= \frac{1}{2\pi} \int_{-w_c}^{w_c} e^{jw\left(t-\frac{\varphi}{w_c}\right)} \, dw$$

$$= \frac{1}{2\pi} \cdot e^{jw\left(t-\frac{\varphi}{w_c}\right)} \cdot \frac{1}{j\left(t-\frac{\varphi}{w_c}\right)} \Bigg|_{-w_c}^{w_c}$$

$$= \frac{1}{2\pi j\left(t-\frac{\varphi}{w_c}\right)} \cdot \left[e^{jw_c\left(t-\frac{\varphi}{w_c}\right)} - e^{-jw_c\left(t-\frac{\varphi}{w_c}\right)} \right]$$

$$= \frac{2j \sin w_c\left(t-\frac{\varphi}{w_c}\right)}{2\pi j\left(t-\frac{\varphi}{w_c}\right)} = \frac{w_c}{\pi} \cdot \frac{\sin(w_c t - \varphi)}{w_c t - \varphi}$$

$$= \frac{w_c}{\pi} \operatorname{sinc}(w_c t - \varphi)$$



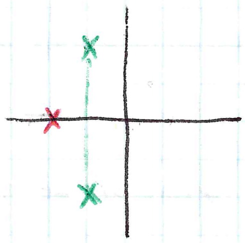
De responsie is al $\neq 0$ voor $t < 0$, dus vóór de oorzaak. Niet te realiseren want niet-causaal

Praktisch gaan we uit van een van de ~~ideale~~ eigenschappen van het ideale filter en proberen die zo goed te benaderen

1. $H(j\omega) = 1$ Butterworth
2. Steile sperflank Tsjebitsjev
   Cauer
3. Lineaire fase-karakteristiek Bessel

P-N beelden

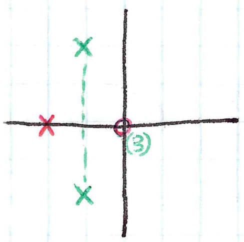
1) L-D



1e orde
3e orde

Een aantal polen in de omgeving van O + evt nulpunten (1 lager dan aantal polen)

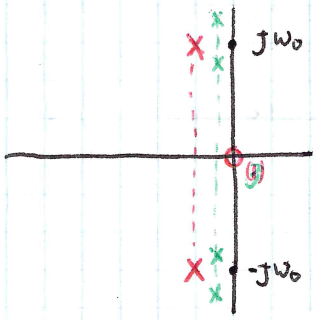
2) H-D



1e orde
3e orde

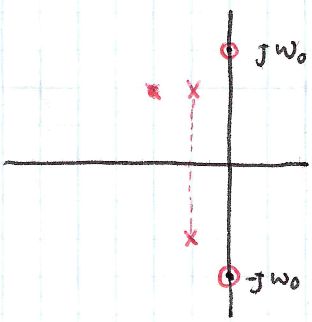
Aantal polen in de omgeving van O + evenveel nulpunten op O

3) B-D



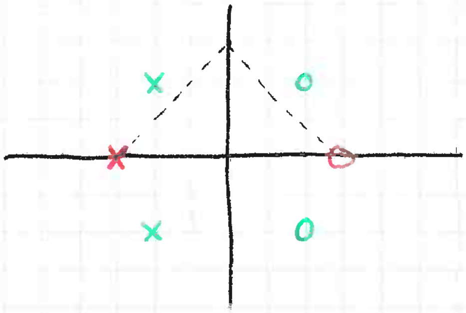
Polen dicht bij centerfreq. ω_0 en minstens 1 nulpunt in O

4) B-S



Een cluster polen bij + en $-j\omega_0$ en evenveel nulpunten in $j\omega_0$ en $-j\omega_0$

5 A-D



Butterworth:

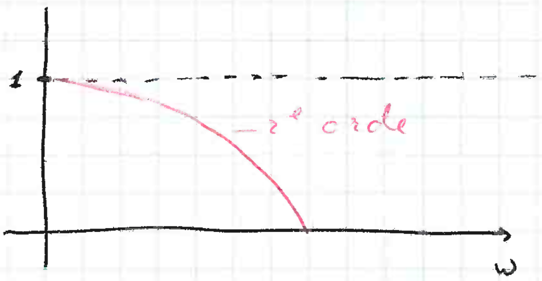
We gaan uit van een laagdoorlaat-filter
We wensen dat $|H(j\omega)|$ in de doorlaatband maximaal vlak is.
Maximaal vlak: { afgezien van de nulde-graadsterm, moet de 1e van 0 verschillende term in de Mc-Lauren-ontwikkeling van ~~mo~~ $|H(j\omega)|$ van zo'n hoog mogelijke graad zijn

***

$$f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots$$

$$= a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

***



1e, 3e, 5e, 7e graads kan niet!

max vlak: zo veel mogelijk afgeleiden moeten 0 zijn

We beschouwen i.p.v $|H(p)|$. $|H(p)|^2 = H(p) \cdot H^*(p)$

$$H(p) = \frac{a_0 + a_1 p + a_2 p^2 + \cdots}{b_0 + b_1 p + b_2 p^2 + \cdots} \rightarrow H(p) \cdot H(p^*)$$

Op de im. as is $H(p^*) = H(-p)$

We bezien verder $H(p) \cdot H(-p)$

Stel $H(p) = \dfrac{T(p)}{N(p)}$! $T = n^e$ graads polynoom

$$T(p) \cdot T(-p) = (a_0 + a_1 p + a_2 p^2 + \cdots) \cdot (a_0 - a_1 p + a_2 p^2 + \cdots)$$

$$= \left(\sum_{i_{even}} a_i p^i + \sum_{i_{oneven}} a_i p^i\right) \cdot \left(\sum_{i_{even}} a_i p^i - \sum_{i_{even}} a_i p^i\right)$$

$$= \left(\sum_{i_{even}} a_i p^i\right)^2 - \left(\sum_{i_{oneven}} a_i p^i\right)^2$$

$$= A_0 + A_1 p^2 + A_2 p^4 + A_3 p^6 + \cdots A_m p^{2m}$$

Iets dergelijks is te schrijven voor $N(p) \cdot N(-p)$

Graad van $T < N$ (laagdoorlaat)

Neem aan: graad van $N = m+1$

$$H(p) \cdot H(-p) = \frac{A_0 + A_1 p^2 + A_2 p^4 + A_3 p^6 + \cdots A_m \cdot p^{2m}}{B_0 + B_1 p^2 + B_2 p^4 + B_3 p^6 + \cdots B_m p^{2m} + B_{m+1} p^{2(m+1)}}$$

$$= C \cdot \frac{1 + \alpha_1 p^2 + \cdots \alpha_m p^{2m}}{1 + \beta_1 p^2 + \cdots \beta_m p^{2m} + \beta_{m+1} p^{2m+2}}$$

$$H(p)\cdot H(-p) = c\cdot\frac{1+\alpha_1 p^2+\dots\alpha_m p^{2m} + (\beta_1 p^2 \dots \beta_{m+1} p^{2m+2}) - (\text{———})}{1+\beta_1 p^2+\dots\beta_{m+1} p^{2(m+1)}}$$

$$= c\cdot\left[1+\frac{(\alpha_1-\beta_1)p^2+(\alpha_2-\beta_2)p^4+\dots(\alpha_m-\beta_m)p^{2m}-\beta_{m+1}p^{2m+2}}{1+\beta_1 p^2+\dots\beta_{m+1}p^{2m+2}}\right]$$

alleen de breuk genoteerd in machten van p

$$\frac{2+\boxed{4}+6+\dots}{0+2+4+6+\dots}$$

diff →
$$\frac{(1+\boxed{3}+5+\dots)(0+2+4\dots)-(2+4+6\dots)(1+3+5\dots)}{(0+2+4+6\dots)^2}$$

$$\frac{1+\boxed{3}+5+\dots}{0+2+4+\dots} \longrightarrow =0 \quad \underline{\text{voor } p=0}$$

diff
$$\frac{(\cancel{0}+\boxed{2}+4+\dots)(0+2+4+\dots)-(\boxed{1}+3+5+\dots)(1+3+5+\dots)}{0+2+4+6\dots \quad \text{nx zo lang}}$$

indien $\underline{\alpha_1-\beta_1}=0$ dan is is afgeleide 0 in $p=0$

$$\frac{\boxed{2}+4+6+\dots}{0+2+4+6+\dots}$$

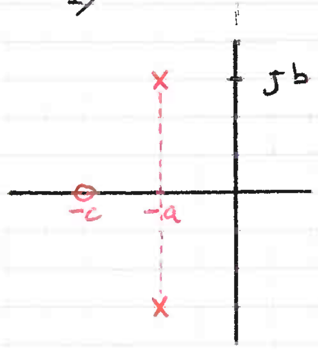
4^e^ afg: $(\alpha_2-\beta_2)=0$

Regel : $\underline{(\alpha_i-\beta_i)=0}$

$1\le i\le m$

Opgaven

1)

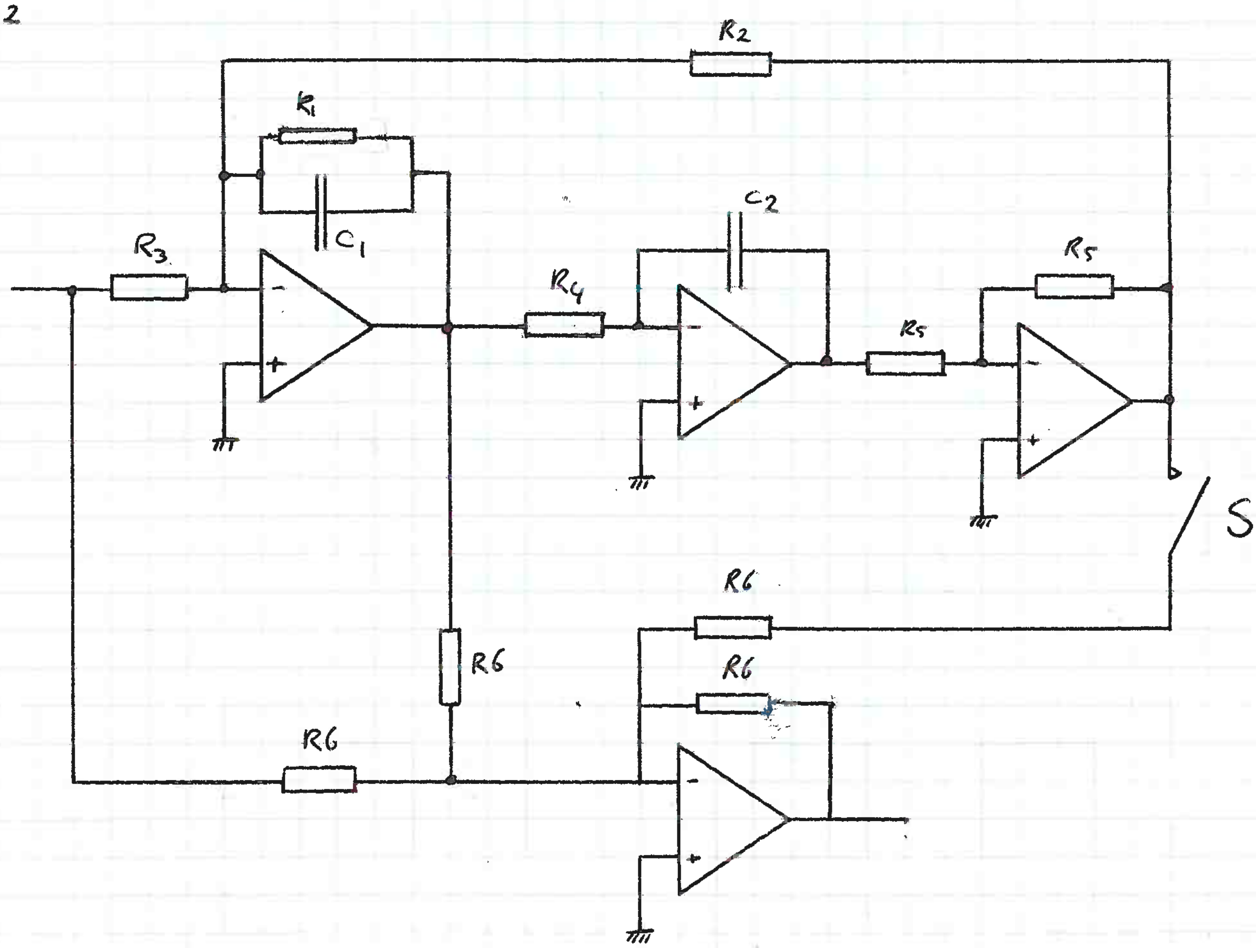


Bepaal c voor maximaal vlakke overdracht,

indien: $a = 1$ en $b = 0$

$a = b$

2



Oplossingen:

$$H(p) = \frac{p+c}{(p+a+jb)(p+a-jb)} = \frac{p+c}{p^2+2ap+(a^2+b^2)}$$

$$H(p)\cdot H(-p) = \frac{p+c}{p^2+2ap+(a^2+b^2)} \cdot \frac{-p+c}{p^2-2ap+(a^2+b^2)}$$

$$= \frac{c^2-p^2}{p^4+p^2(2a^2+2b^2-4a^2)+(a^2+b^2)^2}$$

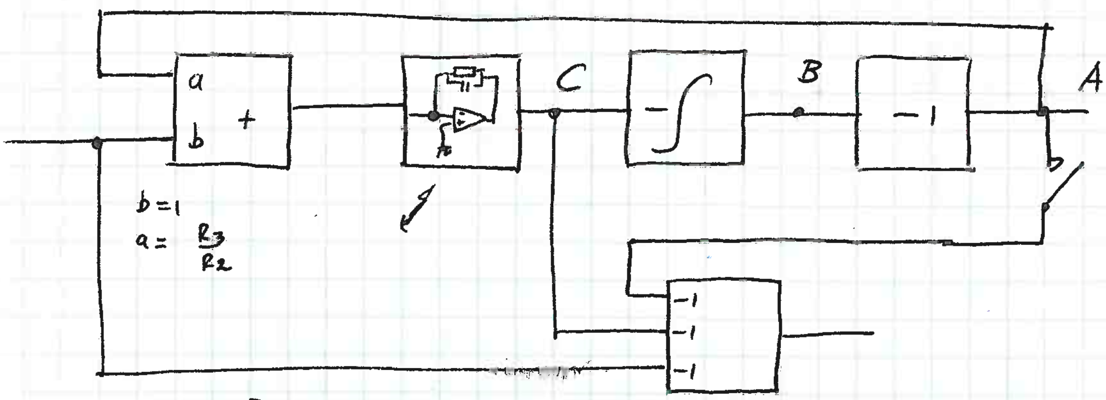
$$= \frac{c^2}{(a^2+b^2)^2} \cdot \frac{1 \underbrace{-\frac{1}{c^2}}_{A_1} p^2}{\underbrace{\frac{p^4}{(a^2+b^2)^2}}_{B_1} + \underbrace{\frac{2(b^2-a^2)}{(a^2+b^2)^2}}_{B_2} p^2 + 1}$$

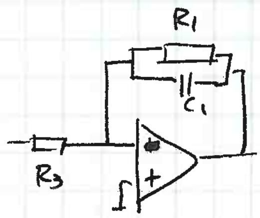
$A_1 = B_1 \longrightarrow$

$$-\frac{1}{c^2} = \frac{2(b^2-a^2)}{(a^2+b^2)^2}$$

$\left.\begin{array}{l} a=1 \\ b=0 \end{array}\right\}$ $\frac{1}{c^2}=2 \longrightarrow c = \frac{1}{2}\sqrt{2}$ $(\text{nulpunt} = -c)$

$a=b \longrightarrow \frac{1}{c^2}=0, \longrightarrow c \longrightarrow \infty$



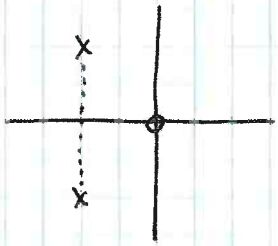


$$H = \frac{-1/R_3C_1}{s+\frac{1}{R_1C_1}}$$

$$U_C = \frac{-\frac{1}{R_3C_1}}{s+\frac{1}{R_1C_1}} \cdot U_{in} - \frac{\frac{1}{R_2C_1}}{s+\frac{1}{R_1C_1}} \cdot U_A$$

$$U_A = \frac{1}{s R_4 C_2} \cdot U_C$$

$$\frac{U_C}{U_{in}} = \frac{-\frac{1}{R_3C_1}\cdot s}{s^2 + \frac{1}{R_1C_1}s + \frac{1}{R_2R_4C_1C_2}} \quad N$$



banddoorlaat

$$\frac{U_b}{U_{in}} = \frac{\frac{1}{R_3R_4C_1C_2}}{N}$$

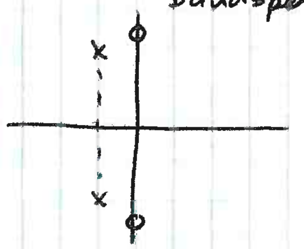
laagdoorlaat

U_D (schak open):

$$\frac{U_d}{U_{in}} = \frac{s^2 + \left(\frac{1}{R_1C_1} - \frac{1}{R_3C_1}\right)s + \frac{1}{R_2R_4C_1C_2}}{N}$$

$R_1 = R_3$:

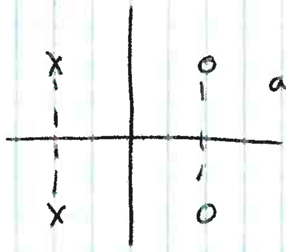
$$\frac{s^2 + \frac{1}{R_2R_4C_1C_2}}{s^2 + \frac{1}{R_1C_1}s + \frac{1}{R_2R_4C_1C_2}} \quad N$$



bandsper.

$R_1 = 2R_3 \Rightarrow$

$$\frac{s^2 - \frac{1}{R_1C_1}s \text{ - - - -}}{s^2 + \frac{1}{R_1C_1}s \text{ - - -}}$$



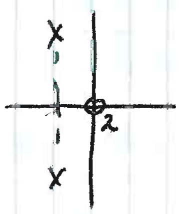
all pass

Ud (schak dicht)

$$\frac{U_d}{U_{in}} = \frac{s^2 + \left(\frac{1}{R_1C_1} - \frac{1}{R_3C_1}\right)s + \frac{1}{R_2R_4C_1C_2} - \frac{1}{R_3R_4C_1C_2}}{N}$$

als $R_1 = R_2 = R_3$

$$\frac{U_d}{U_{in}} = \frac{s^2}{N}$$

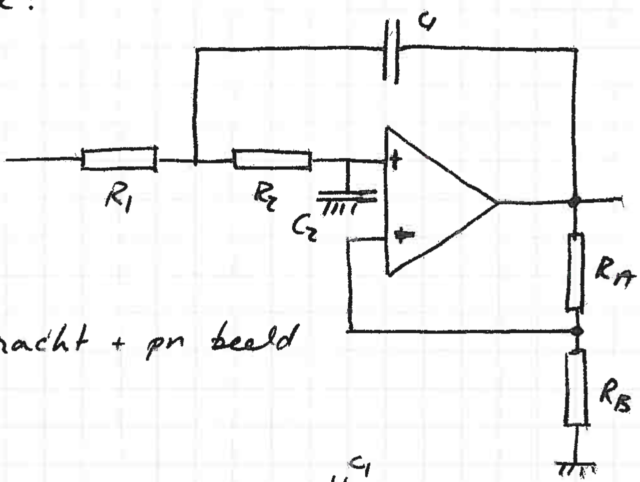


Universele

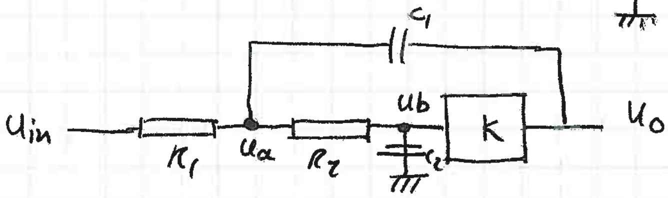
Bikwadratische

Sectie

Sonnetje :



Overdracht + pn beeld



$$H(s) = \frac{K \cdot \frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \left(\frac{1}{R_2 C_2} + \frac{1}{R_2 C_1} + \frac{1}{R_1 C_1} - \frac{K}{R_2 C_2}\right) s + \frac{1}{R_1 R_2 C_1 C_2}}$$

$$K = \frac{R_B + R_A}{R_B}$$

Butterworth: kiest voor de all-pole configuratie:

$$A_1 \text{ t/m } A_m = 0$$

$$\Longrightarrow B_1 \text{ t/m } B_m = 0$$

Dus $H(p) \cdot H(-p) = c \cdot \dfrac{1}{1 + B_n p^{2n}}$

Neem nu reële frequenties:

$$H(j\omega) \cdot H(-j\omega) = c \cdot \frac{1}{1 \pm B_n \omega^{2n}}$$

Neem aan: $\omega_c = \omega_{3dB} = 1$ rad/sec

3 dB $\quad \pm B_n \omega^{2n} = 1$

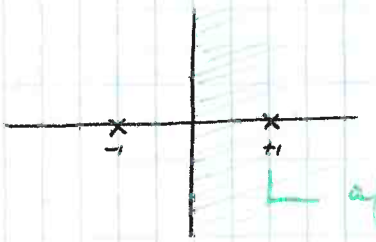
$$B_n = \pm \frac{1}{\omega_c^{2n}} \xrightarrow{\omega_c = 1} B_n = \pm 1$$

Algemeen in LHV:

$$H(p) \cdot H(-p) = c \cdot \frac{1}{1 \pm p^{2n}}$$

c is versterkingsfactor

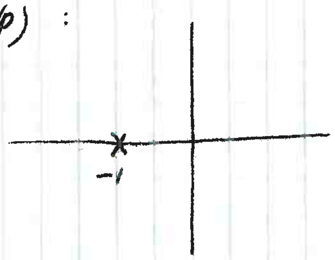
$n = 1$: $\dfrac{1}{1 - p^2} \longrightarrow$ polen: $p = \pm 1$



$\longleftarrow H(p) \cdot H(-p)$

afhankelijk van $H(-p)$

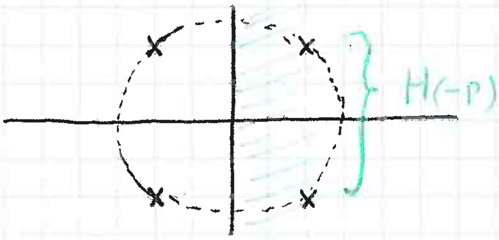
$H(p)$:



$\underline{n=2}$:

$$: \quad \frac{1}{1+p^4} \qquad \text{polen op } p^4 = -1 = e^{j(\pi + k2\pi)}$$

$$p = e^{j(\pi/4 + k\pi/2)}$$



$$H(p) = c \cdot \frac{1}{(p+\frac{1}{2}\sqrt{2}+\frac{1}{2}\sqrt{2}.j)(p+\frac{1}{2}\sqrt{2}-\frac{1}{2}\sqrt{2}.j)}$$

$$= c \cdot \frac{1}{p^2+\sqrt{2}p+1}$$

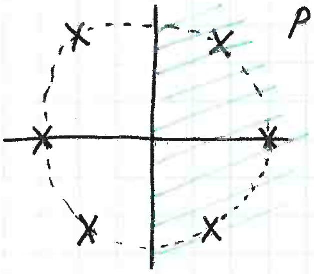
2e graads Butterworth polynoom

$\underline{n=3}$

$$\frac{1}{1-p^6}$$

polen op $p^6 = 1 = e^{jk2\pi}$

$$p = e^{jk.\pi/3}$$



Hiermit volgt:

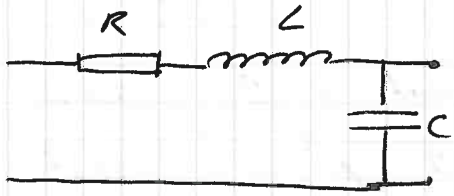
$$H(p) = c\,\frac{1}{p^3+2p^2+2p+1}$$

3e graads

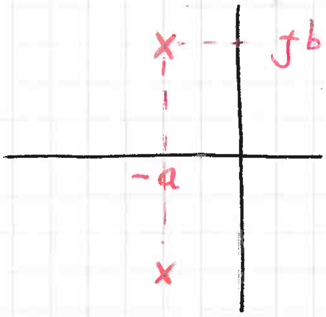
Recept: (n^e orde)

Verdeel $2n$ polen equidistant over de eenheidscirkel met symmetrie t.o.v. Re- en Im-as en geen polen op de Im-as. Schrap vervolgens het interieur van het rechter halfvlak.

Terminologie bij 2e orde filternetwerk



$$H(s) = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L} + \frac{1}{LC}}$$



$$H(s) = \frac{1}{(s+a+jb)(s+a-jb)}$$

$$= \frac{1}{s^2 + 2as + a^2+b^2}$$

$a^2+b^2 = w_0^2 = 1$ (butterworth)

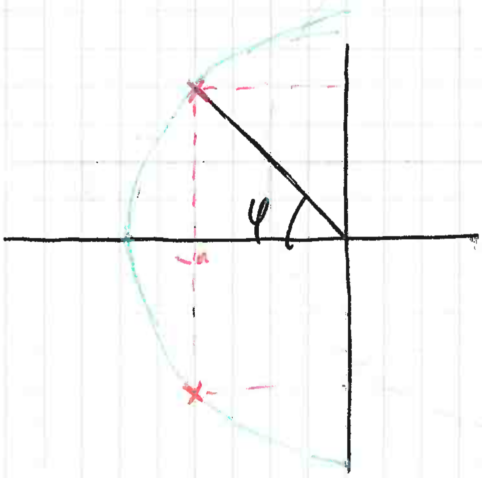
$$H(s) = \frac{1}{s^2 + 2as + 1}$$

nu bij filter : $\frac{1}{LC} = w_0^2$ dus $\frac{R}{L} = \frac{RC}{LC} = w_0^2 RC = \frac{w_0}{Q}$

dus filter : $H(s) = \dfrac{w_0^2}{s^2 + \frac{w_0}{Q} s + w_0^2}$

$w_0 = 1 \rightarrow H(s) = \dfrac{1}{s^2 + \frac{1}{Q} s + 1} \equiv \dfrac{1}{s^2 + 2as + 1}$

$Q = \frac{1}{2a} = \frac{1}{2 \cos\varphi}$



~~Hiert~~

Hiermee is een coördinaten systeem geïntroduceerd:

cirkels met middelpunt in 0 : const w_0

lijnen met constante φ : const. Q

$$\varphi = \frac{\pi}{4} \rightarrow Q = \frac{1}{2 \cdot \frac{1}{2}\sqrt{2}} = \frac{1}{2}\sqrt{2}$$

$$\varphi = \frac{\pi}{2} \qquad Q = \infty$$

$$\varphi = 0 \qquad Q = \frac{1}{2}$$

nu Sallen-Key circuit:

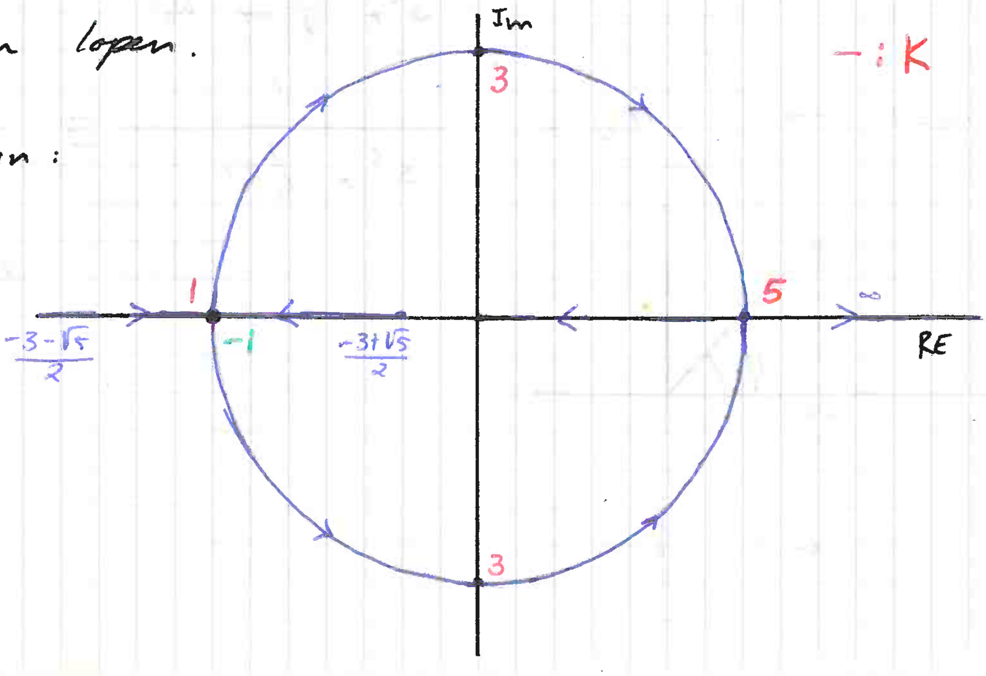
$\underline{w_0 = 1}$ dus $\frac{1}{R_1 R_2 C_1 C_2} = 1$

Te realiseren door $R_1 = R_2 = 1\,\Omega$ & $C_1 = C_2 = 1\,F$

$$H(s) = \frac{k}{s^2 \quad (3-k)s \quad +1}$$

met k-variatie kunnen we de polen over de eenheids-cirkel laten lopen.

De poolbaan:



$$3 - K = \frac{1}{Q}$$

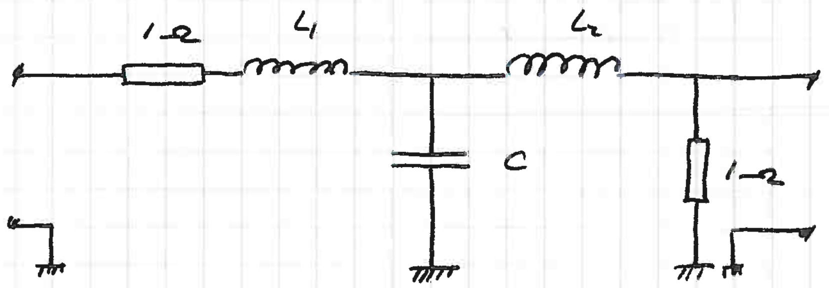
$k \to 0$: $s^2 + 3s + 1 = 0$ $\qquad k \to \infty \quad s = \infty \lor s = 0$

$$s = \frac{-3 \pm \sqrt{5}}{2}$$

Ontwerp **Butterworth** :

1) Ladder passief

vb: 3^e-orde



1 van de weerstanden normeren op 1Ω

$$\begin{pmatrix} 1 & 1+pL_1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ pC & 1 \end{pmatrix}\begin{pmatrix} 1 & pL_2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} =$$

$$H(p) = \frac{1}{(1 + pC + p^2 L_1 C)(1 + pL_2)\{1 + pL_1}$$

$$= \frac{1}{p^3 L_2 L_1 C + p^2 (L_1 C + L_2 C) + p(L_1 + L_2 + C) + 2}$$

$$= \frac{\frac{1}{2}}{p^3 \frac{L_1 L_2 C}{2} + p^2 \left(\frac{L_1 + L_2}{2}\right) C + p \frac{L_1 + L_2 + C}{2} + 1}$$

$$= \frac{1/2}{p^2 + 2p^2 + 2p + 1}$$

3^e graads BW polynoom

dus : $\frac{l_1 l_2 c}{2} = 1 \xrightarrow{c=2} l_1 l_2 = 1$

$$\left.\begin{array}{l} \frac{c(l_1+l_2)}{2} = 2 \\ \\ \frac{l_1+l_2+c}{2} = 2 \end{array}\right\} \quad c(l_1+l_2) = c + l_1 + l_2 = 4.$$

$c = 2$ en $l_1 + l_2 = 2$

uit $l_1 l_2 = 1$ en $l_1 + l_2 = 2$ volgt

$$l_1 = l_2 = 1$$

Denormeren:

R : $1\,\Omega$ naar $R\,\Omega$:

alle impedantie met R te vermenigvuldigen

$L \to RL$ $\quad c \to \frac{c}{R}$

W_c : 1 rad/s $\longrightarrow$ W_c rad/se

reactieve comp

alle comp. door W_c delen.

$L \to \frac{L}{W_c}$

$c \to \frac{c}{W_c}$

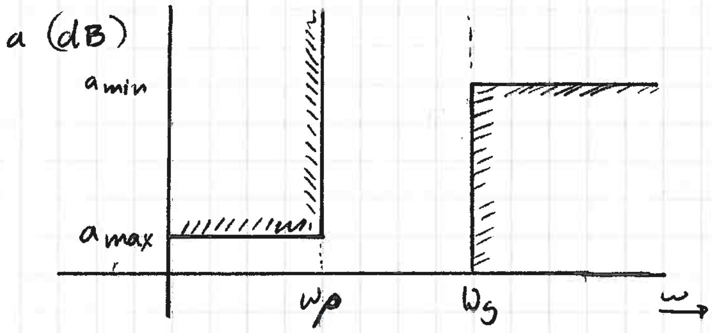
Stel $R = 1k\Omega$ en $f_c = 10\ kHz$

$$L_1 = L_2 = \frac{1 \cdot 10^3}{2\pi \cdot 10^4} =$$

$$C = \frac{2}{10^3 \cdot 2\pi \cdot 10^4} =$$

Hogere orde ladders: <u>tabellen</u> <u>gebruiken</u>

Praktisch wordt een filter niet in eerste instantie door de orde gespecificeerd maar als volgt:



Butterworth: $$|H(j\omega)|^2 = \frac{1}{1 + \left(\frac{\omega}{\omega_c}\right)^{2n}}$$

$$-a = 20\ {}^{10}\log |H(j\omega)|$$

$$= 20 \cdot {}^{10}\log \left[1 + \left(\frac{\omega}{\omega_c}\right)^{2n}\right]^{-\frac{1}{2}}$$

$$a = \boxed{10\ {}^{10}\log \left[\left(1 + \frac{\omega}{\omega_c}\right)^{2n}\right]} \quad (2)$$

$$\left(\frac{\omega}{\omega_c}\right)^{2n} = 10^{\frac{a}{10}} - 1 \qquad \boxed{\frac{\omega}{\omega_c} = \left[10^{\frac{a}{10}} - 1\right]^{+\frac{1}{2n}}} \quad 1)$$

ω_p, a_{max}:

$$\frac{\omega_p}{\omega_c} = \left[10^{\frac{a_{max}}{10}} - 1\right]^{\frac{1}{2n}}$$

ω_s, a_{min}

$$\frac{\omega_s}{\omega_c} = \left[10^{\frac{a_{min}}{10}} - 1\right]^{\frac{1}{2n}}$$

deze twee delen: w_c valt eruit

$$\frac{w_p}{w_s} = \left[\left(10^{\frac{a_{max}}{10}}-1\right) / \left(10^{\frac{a_{min}}{10}}-1\right)\right]^{\frac{1}{2n}}$$

$$\boxed{n = \frac{{}^{10}\log\left(10^{\frac{a_{max}}{10}}-1\right) / \left(10^{\frac{a_{min}}{10}}-1\right)}{2\ {}^{10}\log \frac{w_p}{w_s}}}$$

Vb: $a_{max} = 0{,}5$ dB

$a_{min} = 20$ dB

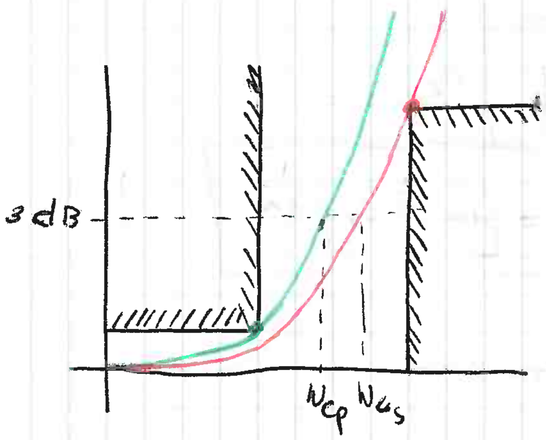
$w_p = 1000$ rad/sec

$w_s = 2000$ rad/sec

$$n = \frac{{}^{10}\log\left[\left(10^{0{,}05}-1\right) / \left(10^{2}-1\right)\right]}{2 \cdot {}^{10}\log \frac{1}{2}} = 4{,}83$$

afronden: $n = 5$

Er zijn na afronding dus meer mogelijkheden om aan de eisen valdoen



w_p, a_{max}

uit ① volgt:

$$w_{c_p} = 1000 \cdot \left[10^{0{,}05}-1\right]^{-\frac{1}{10}}$$

$$= 1234 \text{ rad/s}$$

$$a_{ws} = 10\ {}^{10}\log\left[1+\left(\frac{2000}{1234}\right)^{10}\right] = 21 \text{ dB}$$

— ω_s , a_{min}

$$\omega_{c_5} = 2000 \left[10^{2}-1\right]^{-\frac{1}{10}} = 1263 \text{ rad/s.}$$

$$a_{wp} = 10 \; {}^{10}\log \left[1 + \left(\frac{1000}{1263}\right)^{10}\right] = 0{,}4 \text{ dB}$$

Nu oplossing verder uitwerken met een cascade van Sallen-Key filters

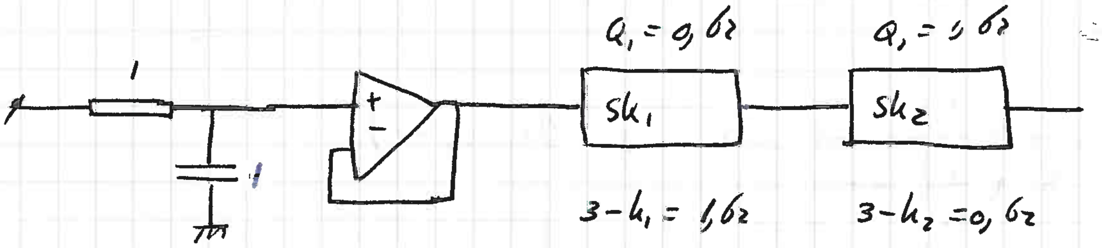
Overdracht :

$$H(s) = \frac{K_t}{(s+1)\left(s^2 + \frac{1}{Q_1}s + 1\right)\left(s^2 + \frac{1}{Q_2}s + 1\right)}$$

$\rightarrow 0{,}618033988$

$$Q_1 = \frac{1}{2\cos\frac{\pi}{5}} = 1{,}62 \qquad Q_2 = \frac{1}{2\cos\frac{2\pi}{5}} = 0{,}62$$

$$\frac{K_t}{(s+1)\,(s^2 + 0{,}62\,s + 1)(s^2 + 1{,}62\,s + 1)}$$



$3 - k_1 = 1{,}62$ $\qquad$ $3 - k_2 = 0{,}62$

$k_1 = 1{,}38$ $\qquad$ $k_2 = 2{,}38$

denormering

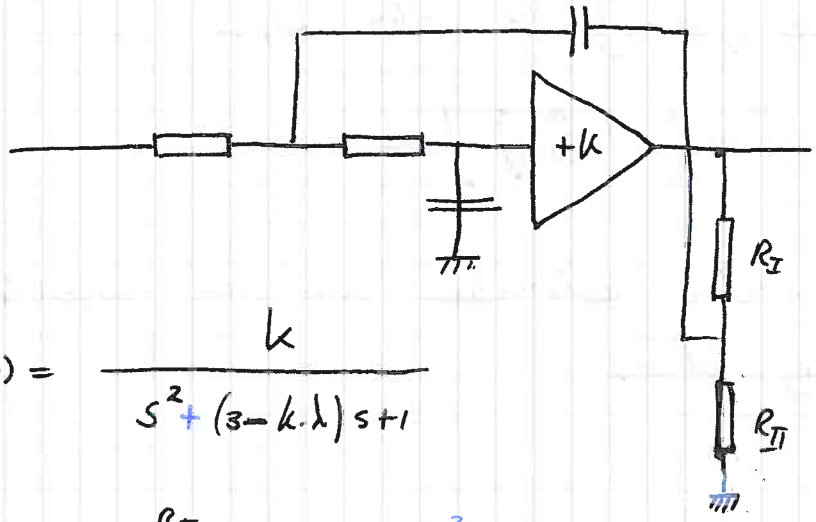
neem aan: eerste $R = 1\,k\Omega$

dan alle R's $1\,k\Omega$

en alle c's : $\frac{1}{1250 \cdot 10^3} = 0{,}8 \cdot 10^{-6}\,F = 0{,}8\,\mu F$

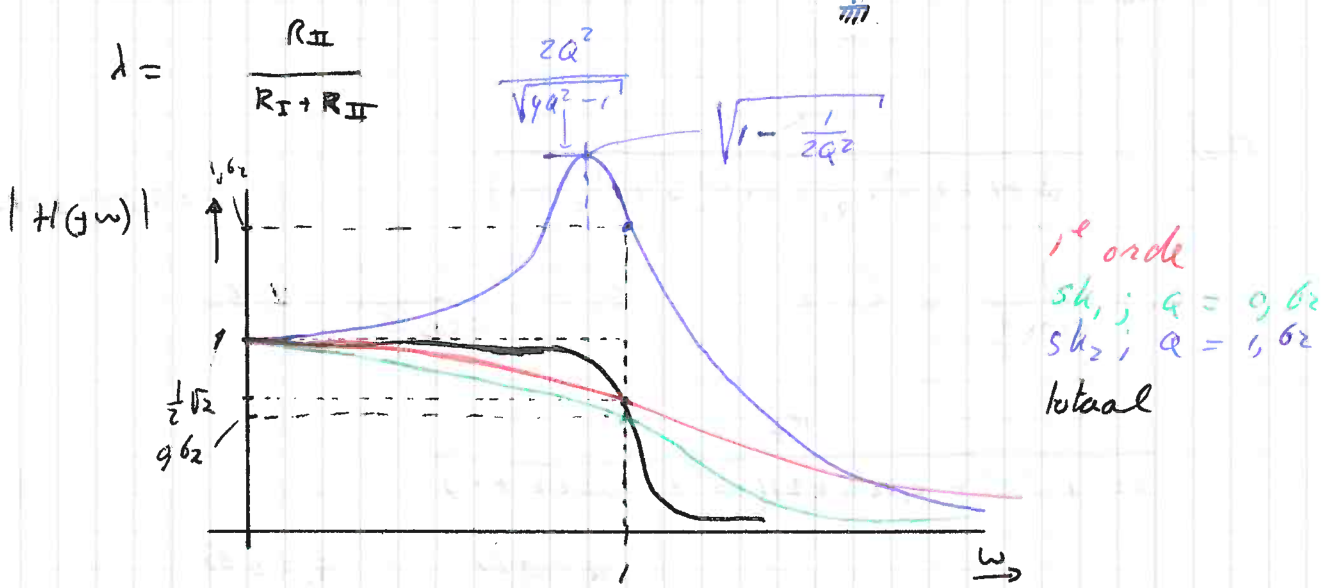
$K_t = 1{,}38 \times 2{,}38 = 3{,}28$

k_t = te vergroten door 1 of beide Sallen-Key resistive gain enhancement toe te passen



$$H(s) = \frac{k}{s^2 + (3 - k\cdot\lambda)s + 1}$$

$$\lambda = \frac{R_{II}}{R_I + R_{II}}$$



$$H(s) = \frac{1}{s^2 + \frac{1}{Q}s + 1} \quad \text{voor } s = j \quad (\omega = \omega_c = 1)$$

$$\frac{1}{-1 + \frac{j}{Q} + 1} \rightarrow |H(j)| = Q$$

proefwerk 27 nov 1987 E4C

a)

$$① \quad \left[U_1 \left\{\frac{1}{R_1}\right\} + Ub \left\{\frac{1}{R_4}\right\} \right] \left[\frac{-R_2}{1+pR_2C_2} \right] = U_A$$

$$② \quad Ub = \frac{10}{pR_3C_3} \cdot U_A$$

$$② \text{ in } ①: \quad \left[U_1 \left\{\frac{1}{R_1}\right\} + U_a \left\{\frac{10}{pR_3C_3R_4}\right\} \right] \frac{-R_2}{1+pR_2C_2} = U_A$$

$$U_A \left[\frac{-10R_2}{pR_cC_3R_4(1+pR_2C_2)} - 1 \right] = U_1 \frac{1}{R} \cdot \frac{+R_2}{1+pR_2C_2}$$

$$\frac{U_a}{U_1} = \frac{-R_2/R_1}{-10R_2 - pR_3C_3R_4(1+pR_2C_2)} \cdot \frac{pR_3C_3R_4 \cdot}{1}$$

$$= + \frac{\frac{pR_2R_3R_4C_3}{R_1}}{p^2R_2R_3R_4C_2C_3 + pR_3C_3R_4 + 10R_2}$$

$$= \frac{\frac{p}{C_2R_1}}{p^2 + p\frac{1}{R_2C_2} + \frac{1}{R_3R_4C_2C_3}} \qquad \text{banddoorlaat}$$

---

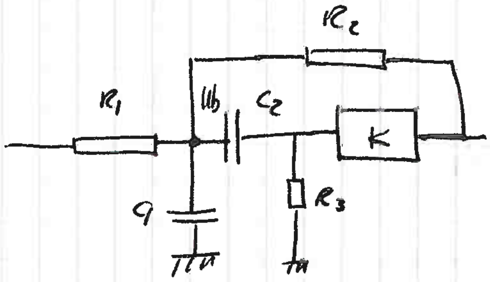
weten:

$$Z_a = \sqrt{Z_{k_1} \cdot Z_{n_1}}$$
$$Z_b = \sqrt{Z_{k_2} \cdot Z_{n_2}}$$

~~Opgave~~

proefwerk 15 nov 1985 E4C

3



$$\frac{U_b - U_1}{R_1} + U_b\, pC_1 + \left(U_b - \frac{U_2}{K}\right)(pC_2) + \frac{U_b - U_2}{R_2}$$

$$-\left(U_b - \frac{U_2}{K}\right) pC_2 + \frac{U_2}{K}\frac{1}{R_3} = 0$$

$$U_b\{1 + p + p + 1\} + U_1\{-1\} + U_2\left\{-\frac{p}{K} - 1\right\} = 0$$

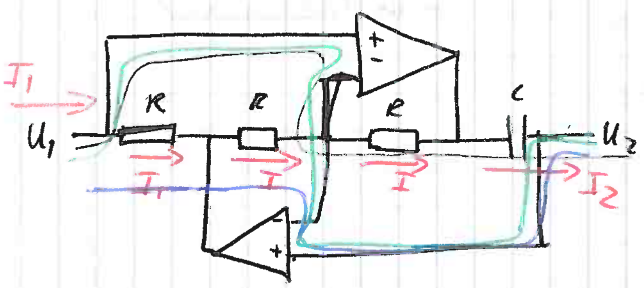
$$-U_b\, p + U_2\left\{\frac{p}{K} + \frac{1}{K}\right\} = 0.$$

$$\left\{\frac{1}{K} + \frac{1}{Kp}\right\} \cdot U_2 \cdot \{2 + 2p\} - U_1 - U_2\left(1 + \frac{p}{K}\right) = 0$$

$$U_2\left[\frac{1}{K}\left(1 + \frac{1}{p}\right)(2 + 2p) - 1 - \frac{p}{K}\right] = U_1.$$

$$\Rightarrow \quad \frac{U_2}{U_1} = \frac{pk}{p^2 + (4 - k)p + 2}$$

$R = 1 \quad C = 1$



GIC

$U_1 = U_2$

blauw $\quad U_1 - J_1 - J = U_2$

rood $\quad U_1 - I - \frac{I_2}{p} = U_2$

$$I_1 + I = I + \frac{I_2}{p}$$

$$I_1 = I_2 / p \quad \rightarrow \quad \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{p} \end{pmatrix}$$

proefwerk 15 nov '05 vraag 2 (volg.)

$$(B) = p\begin{pmatrix} \frac{1}{p} & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}$$

Belasten met condensator C

$$\begin{pmatrix} 1 & 0 \\ 0 & p \end{pmatrix}\begin{pmatrix} 1 & 0 \\ pC & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ p^2C & p \end{pmatrix}$$

$$Z_{in} = \frac{1}{p^2C} \quad : p = j\omega \longrightarrow Z_{in} = -\frac{1}{\omega^2 C}$$

<u>FDNR</u> frequentie dependent negative resistor

<u>Hoogdoorlaat</u>: Ontstaat uit LD door in het complexe vlak 0 en het punt op ∞ te verwisselen, dus spiegelen in eenheidscirkel

D.W.Z: $\underline{\underline{p \to \frac{1}{p}}}$

Bij Butterworth blijven de polen op de eenheidscirkel en er ontstaan bij het n^e orde filter n nulpunten in 0

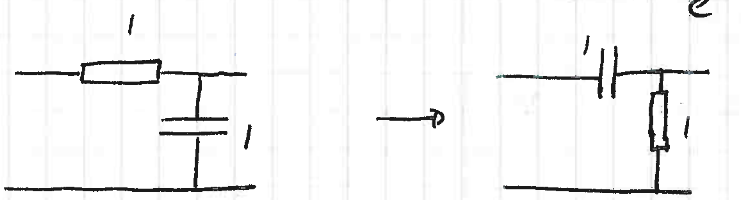
Vb 2^e orde LD: $\frac{1}{p^2+\sqrt{2}p+1} \xrightarrow{HD} \frac{1}{\frac{1}{p^2}+\sqrt{2}\frac{1}{p}+1} = \frac{p^2}{p^2+\sqrt{2}p+1}$

ladderfilter $pL \to \frac{1}{p}L = \frac{1}{p\frac{1}{L}}$, dus cap $\frac{1}{L}$

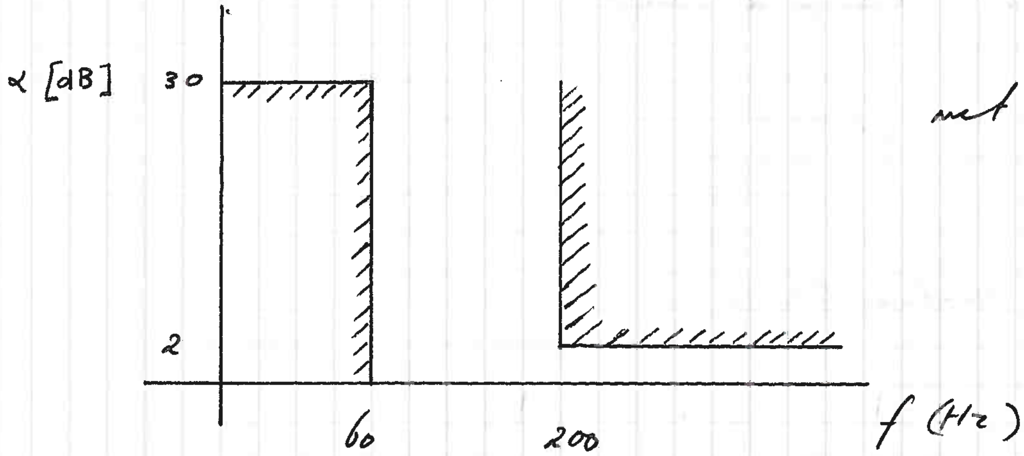
$\frac{1}{pC} \to \frac{1}{\frac{1}{p}C} = p\frac{1}{C}$, dus zelfind $\frac{1}{C}$

RC filter : R $\to$ cap $\frac{1}{R}$

C $\to$ weerst $\frac{1}{C}$



opgave Ontwerp filter met spec's :



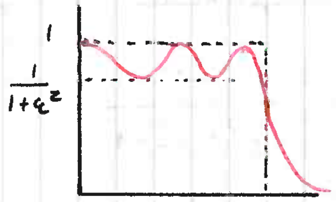
met $C = 0{,}1\,\mu F$

Tsjebitsjev Filters :

Teneinde een steilere sperflank te krijgen, zien we af van een monotoon verloop van a in de doorlaatband; we staan een rimpel toe (equal ripple)

BW: $|H(j\omega)|^2 = \dfrac{k}{1+\left(\frac{\omega}{\omega_0}\right)^{2n}}$, we zoeken een functie verzameling met :

$$|H(j\omega)|^2 = \frac{k}{1+F_n^2(\omega)}$$, equal ripple tussen K en $\dfrac{k}{1+\epsilon^2}$



Dus :

$$1 \leq 1+F_n^2(\omega) \leq 1+\epsilon^2$$

$$0 \leq F_n^2(\omega) \leq \epsilon^2$$

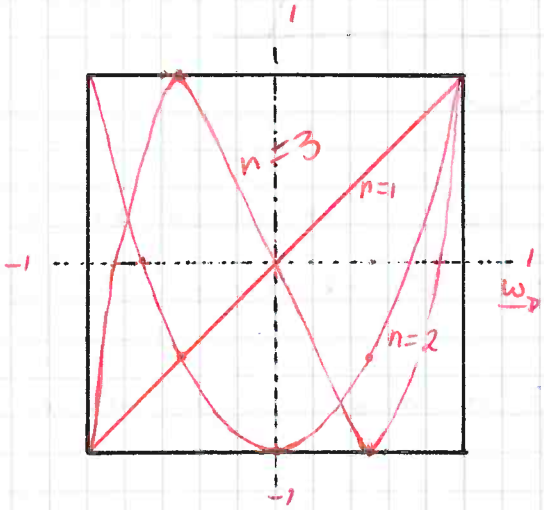
$$0 \leq \frac{F_n^2(\omega)}{\epsilon^2} \leq 1$$

↓ wortel

$$-1 \leq \frac{F_n(\omega)}{\epsilon} \leq +1 \quad \text{op } [-1, 1]$$

We passen Lissajous-figuren toe : $x = \cos kt$
$y = \cos n.k.t$

$\Delta\varphi = 0 \rightarrow$ open figuren



dus $y = \cos(n \arccos x)$

$n = 0 \rightarrow y = 1$

$n = 1 \rightarrow y = \underline{x}$

$n = 2 \rightarrow y = \cos(2 \arccos x)$

$y = 2\cos^2(\arccos x) - 1$

$y = 2x^2 - 1$

$n = 3 \rightarrow y = \cos(3 \arccos x)$

$= 2(2x^2-1).x - x = 4x^3 - 3x$

$= 2\cos(2\arccos x)\cos(\arccos x) - \cos(\arccos x)$

---

algemeen :

$\cos[\{n+1\}z] = \cos nz \cos z - \sin nz \sin z$

$= 2\cos nz \cos z - (\cos nz \cos z + \sin nz \sin z)$

$\cos((n+1)z) = 2\cos nz \cos z - \cos((n-1)z)$

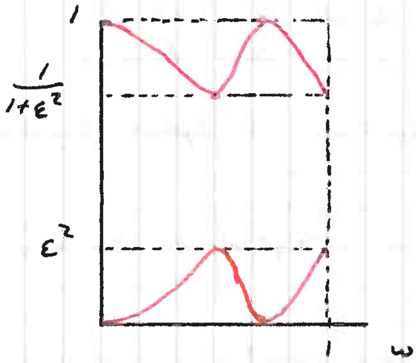
$= 4 \rightarrow y = \cos(4 \arccos x)$

$= 2(4x^3 - 3x)x - (2x^2 - 1) = 8x^4 - 8x^2 + 1$

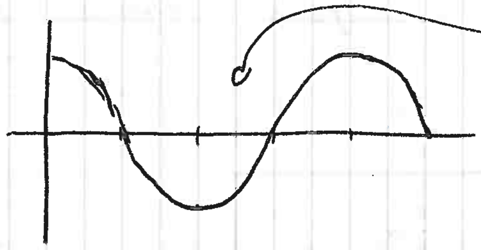
De zo verkregen polynomen heten Tsjebitsjev polynomen
(Let op: deze hebben een heel andere rol dan BW polynomen!)
Na kwadrateren, vermenigvuldigen met ε^2 en opschuiven naar $|H(j\omega)|^2 = 1$ onstaat de Tsjebitsjev overdracht :

$$|H(j\omega)|^2 = \frac{1}{1 + \varepsilon^2 T_n^2(\omega)}$$

waarin $T_n(\omega)$ de n^e graads Tsj. polynoom is bv $n = 3$



uitrekenen : vb : $\cos(3\,\underbrace{(\arccos x)})$



$y = 0 \rightarrow \cos(90°) \rightarrow \arccos x = 30° \quad x = \frac{1}{2}\sqrt{3}$

$y = 1 \rightarrow \cos(0°) \rightarrow \arccos x = 0° \quad x = 1$

bedrag $T_n(\omega)$ in spergebied $\cos(n \arccos \omega)$ voor $\omega > 0$

voor $\omega > 1$ $\quad \arccos \omega = jz$

$$\omega = \cos jz = \frac{e^{j(jz)} + e^{-j(jz)}}{2} = \frac{e^{-z} + e^{z}}{2} = \cosh z$$

dus ~~voor~~ :

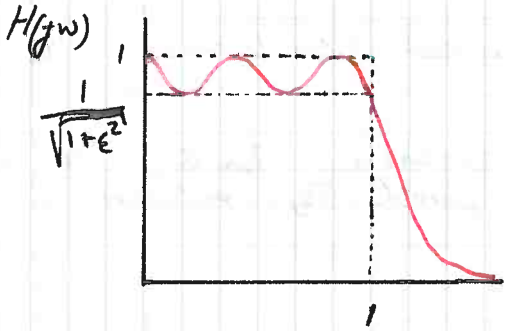
$$\left.\begin{aligned} T_n(\omega) &= \cos(n \arccos \omega) \\ T_{n+1}(\omega) &= 2\omega\, T_n(\omega) - T_{n-1}(\omega) \end{aligned}\right\} \underline{|\omega| \leq 1}$$

~~voor~~ $\arccos \omega = j\, \text{areacosh}\, \omega$

dus
$$\left.\begin{aligned} T_n(\omega) &= \cos(nj\, \text{areacosh}\, \omega) \\ T_n(\omega) &= \cosh(n\, \text{areacosh}\, \omega) \end{aligned}\right\} \underline{\omega > 1}$$

voor bv $3 = n$ geldt :

$$T_n = \cos(3 \arccos \omega) = 4\omega^3 - 3\omega \quad \text{voor } \omega \leq 1$$
$$= \cosh(3\, \text{areacosh}\, \omega) = 4\omega^2 - 3\omega \quad \text{voor } \omega > 1$$



Aantal maxima bepaalt orde

$\underline{\text{Orde} = 5}$

$T_n(\omega)$ voor $\omega > 1$ is monotoon stijgend (cosh monotoon stijgend!)
Een aantal T_n's in polynoom vorm:

$$T_1(\omega) = \omega$$
$$T_2(\omega) = 2\omega^2 - 1$$
$$T_3(\omega) = 4\omega^3 - 3\omega$$
$$T_4(\omega) = 8\omega^4 - 8\omega^2 + 1$$
$$T_5(\omega) = 16\omega^5 - 20\omega^3 + 5\omega$$

Voor n oneven: $T_n(0) = 0$ dwz $|H(0)| = 0$

Voor n even: $T_n(0) = \pm 1$ dwz $|H(0)| = \frac{1}{\sqrt{1+\varepsilon^2}}$

zie $|H(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2 T_n^2(\omega)}}$

Voor $\omega = 1$ $T_n(\omega) = 1$

$$|H(j\omega)| = \frac{1}{\sqrt{1+\varepsilon^2}}$$

Waar liggen de polen van het T-filter

Stel op: $\sigma + j\omega$: Dan volgt na enig rekenwerk:

$$\left.\begin{aligned} \omega &= \cos\mu \cdot \cosh\nu \\ \sigma &= -\sin\mu \cdot \sinh\nu \end{aligned}\right\} \text{ met } \mu = \frac{2k+1}{n}\cdot\frac{\pi}{2} \;;\; \nu\ \frac{1}{n}\,\text{areasinh}\,\frac{1}{\varepsilon}$$

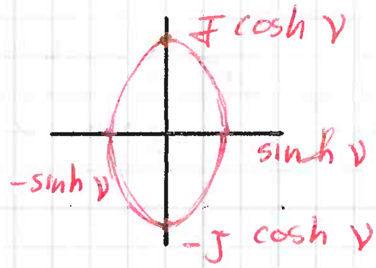
Hieruit volgt:

$$\left(\frac{\omega}{\cosh\nu}\right)^2 + \left(\frac{\sigma}{\sinh\nu}\right)^2 = \cos^2\mu + \sin^2\mu = 1 \qquad \text{ellips } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

De polen liggen op een ellips! (vgl Butterworth op cirkel)

Snijpunt Im-as: $\omega = \pm \cosh\nu$

Re-as: $\sigma = \pm \sinh\nu$



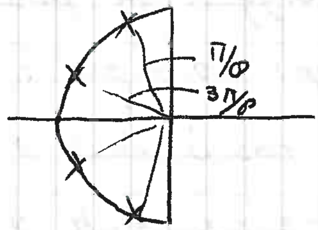
Lange as langs Im. as

Vergelijken we de poolposities van T met die van BW.

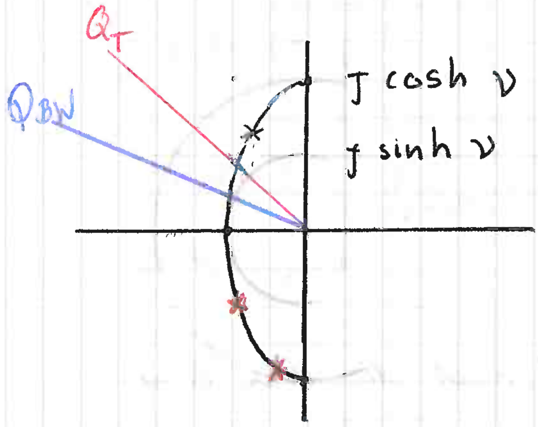
neem als vb: $n=4$

T	BW
$k=0 \rightarrow \mu = \frac{\pi}{8}$	
$\left.\begin{aligned} \omega &= \cos\frac{\pi}{8}\cdot\cosh\nu \\ \sigma &= -\sin\frac{\pi}{8}\cdot\sinh\nu \end{aligned}\right\}$ $\longleftrightarrow$	$\left\{\begin{aligned} \omega &= \cos\frac{\pi}{8}\cdot 1 \\ \sigma &= -\sin\frac{\pi}{8}\cdot 1 \end{aligned}\right.$
$k=1 \rightarrow \mu = \frac{3\pi}{8}$	
$\left.\begin{aligned} \omega &= \cos\frac{3\pi}{8}\cosh\nu \\ \sigma &= -\sin\frac{3\pi}{8}\sinh\nu \end{aligned}\right\}$ $\longrightarrow$	$\left\{\begin{aligned} \omega &= \cos\frac{3\pi}{8}\cdot 1 \\ \sigma &= -\sin\frac{3\pi}{8}\cdot 1 \end{aligned}\right.$

rest polen af te leiden
2 complex toegevoegd +
4 gespiegelde
↓
niet meer k's



De reële en imaginaire delen van de T-polen worden bepaald zoals bij BW, maar we gebruiken nu cirkels met de straal $\cosh\nu$ resp $\sinh\nu$, ipv straal 1



<u>vergelijking BW – T's</u>

1) de polen van T hebben grotere Q dan die van BW bij gelijke n.

Tabel voor $n=5$

N	BW	0,5	1	3
		T's		
$\frac{\pi}{10}$	1,62	4,54	5,5	8,81
$\frac{3\pi}{10}$	0,62	1,18	1,306	2,137
$\frac{5\pi}{10}$	0,5	0,5	0,5	0,5

← rimpel in dB

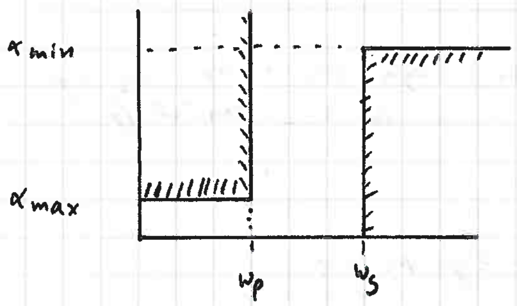
2) De groeplooptijd t_g is bij T's minder constant (grotere looptijdvervorming)

3) T heeft steilere sperflank

4) Voor dezelfde specificatie is $n_{T_s} < n_{B_w}$ (niet afgerond!)

5) T heeft grotere demping diep in het spergebied

Ontwerp van een Tsjebitsjev filter



$a_{max} \longrightarrow$ rimpel

Op w_p normeren

$$a = -20 \, {}^{10}\log |H(jw)| = -20 \, {}^{10}\log \frac{1}{\sqrt{1+\varepsilon^2 T_n^2(w)}}$$

$$= 10 \, {}^{10}\log \left(1 + \varepsilon^2 T_n^2(w)\right)$$

Stel $w_p = 1$ In $w=1$ geldt $T_n(w) = 1$

$$a_{max} = 10 \, {}^{10}\log (1+\varepsilon^2) \longrightarrow \varepsilon = \sqrt{10^{\frac{a_{max}}{10}} - 1}$$

Tabel :

α_m [dB]	ε
0,1	0,152
0,5	0,349
1	0,508
2	0,765
3	0,998

Nu de orde van het filter bepalen

Voor w_s: $\alpha_{min} = 10\ {}^{10}\log\left(1+\varepsilon^2 T_n^2(w_s)\right)$

$\alpha_{min} = 10\ {}^{10}\log\left(1+\varepsilon^2\cosh^2(n \text{ areacosh } w_s)\right).$

opm: we zijn buiten het doorlaat gebied dus cos → cosh

$$\cosh^2(n \text{ areacosh } w_s) = \frac{\left(10^{\frac{\alpha_{min}}{10}}-1\right)}{\varepsilon^2} = \frac{\left(10^{\frac{\alpha_{min}}{10}}-1\right)}{\left(10^{\frac{\alpha_{max}}{10}}-1\right)}$$

$$n = \frac{\text{areacosh}\left\{\left[10^{\frac{\alpha_{min}}{10}}-1\right]/\left[10^{\frac{\alpha_{max}}{10}}-1\right]\right\}^{\frac{1}{2}}}{\text{areacosh } w_s}$$

Voorbeeld: $\alpha_{max} = 0{,}5$ dB $\quad \alpha_{min} = 22$ dB $\quad f_p = 1$ kHz $\quad f_s = 2\frac{1}{3}$ kHz

ε: zie tabel = 0,349

$$n = \frac{\text{areacosh}\left\{\left[10^{2,2}-1\right]/\left[10^{0,05}-1\right]\right\}^{\frac{1}{2}}}{\text{areacosh } 2{,}33} = 2{,}87 \rightarrow 3$$

eigenlijk moet dit w_s zijn maar mag f_s zijn omdat $f_p = 1$

*** areacosh:

$$\frac{e^x + e^{-x}}{2} = \cosh x = y \rightarrow e^{2x} - 2y + 1 = 0$$

$$e^x = \frac{2y \pm \sqrt{4y^2 - 4}}{2}$$

$$= y \pm \sqrt{y^2 - 1}$$

$$x = \ln\left(y \pm \sqrt{y^2-1}\right)$$

***

Vergelijk Butterworth met dezelfde specificaties

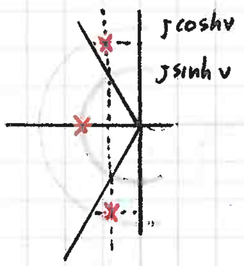
$$n_{Butter} = \frac{\log \ \text{---}}{2\log \ \text{--}} = 4{,}23 \rightarrow 5$$

Bepalen van poolposities (ν)

$\nu = \frac{1}{n}$ areasinh $\frac{1}{\varepsilon} = \frac{1}{3}$ areasinh $\frac{1}{0{,}349} = 0{,}592$

$\cosh \nu = \cosh 0{,}592 = 1{,}18$

$\sinh \nu = 0{,}627$



$$H(s) = \frac{1}{(s+0{,}627)(s+0{,}627\overbrace{\cos\tfrac{\pi}{3}}^{=\sin \pi/6} + j\,1{,}18\cos\tfrac{\pi}{6})(s+0{,}627\cos\tfrac{\pi}{3} - j\,1{,}18\cos\tfrac{\pi}{6})}$$

$$= \frac{1}{(s+0{,}627)(s+0{,}314+j\,1{,}022)(s+0{,}314-j\,1{,}022)}$$

$$= \frac{1}{(s+0{,}627)\underbrace{(s^2+0{,}628\,s+1{,}143)}_{s^2+\frac{\omega_0}{Q}\cdot s+\omega_0^2}}$$

$\omega_0 = \sqrt{1{,}143} = \underline{1{,}069}$

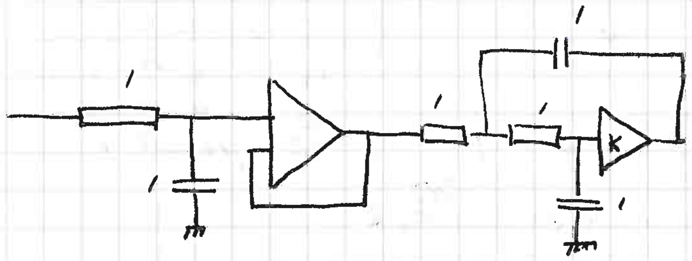
$0{,}628 = \frac{1{,}069}{Q} \rightarrow Q = \underline{1{,}702}$

Voor een tweede-orde overdracht H(s) geldt per μ:

$$H(s) = \frac{1}{s^2 + 2\sinh\nu\sin\mu\, s + (\sinh\nu\sin\mu)^2 + (\cosh\nu\cos\mu)^2}$$

$\frac{\omega_0}{Q} = 2\sinh\nu\sin\mu$

$\omega_0^2 = (\sinh\nu\sin\mu)^2 + (\cosh\nu\cos\mu)^2$



0,627 1,069 Denormering naar gewenste 3 dB frequentie ($\omega_p = 1$)

Daarna naar 1kHz normeren

→ $2\pi\, 10^3$ $2\pi \cdot 10^3$

Daarna denormeren naar $R = 1\,k\Omega$

10^3 10^3

↓ ↓

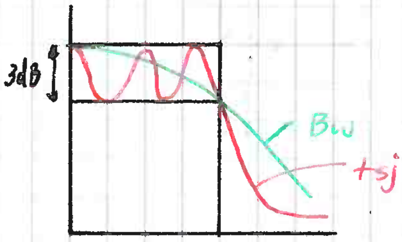
$C = \frac{1}{0{,}627 \cdot 2\pi \cdot 10^3 \cdot 10^3} = 254$ nF $C'_s = \frac{1}{1{,}069 \cdot 10^3 \cdot 10^3 \cdot 2\pi} = 149$ nF

$a_{max} = 0{,}3\ dB \qquad a_{min} = 22\ dB \qquad W_p = 2\pi . 10^4 \qquad \omega_s = 24{,}6 . 2\pi . 10^3$

gebruik SK's met $K=1$ en $R = 1{,}5\,k\Omega$

<u>Vgl : BW − Tsj</u>

1) Damping diep in het spergebied



Neem Rimpel 3 dB

$$BW : |H(j\omega)| = \frac{1}{\sqrt{1+\omega^{2n}}}$$

$$a = 10\ {}^{10}\log\left(1+\omega^{2n}\right)$$

voor $\omega >> 1 \qquad a \simeq 20\,n\ {}^{10}\log \omega$

Tsj : $a = 10\ {}^{10}\log\left(1+\varepsilon^2 T_n^2\, \omega\right) \qquad \varepsilon = 1$ (rimpel 3dB)

$T_1 = \omega$
$T_2 = 2\omega^2 - 1$
$T_3 = 4\omega^3 - 3\omega$
$T_4 = 8\omega^4 - 8\omega^2 + 1$
$T_5 = 16\omega^5 + \dots$

voor $\omega >> 1 \qquad T_n \simeq 2^{n-1} . \omega^n$

$$a \simeq 10\ {}^{10}\log\left(2^{2n-2} . \omega^{2n}\right)$$
$$= 20\,n\ {}^{10}\log \omega + 10(2n-2)\ {}^{10}\log 2$$
$$+ \underline{\underline{6{,}02\,(n-1)}}$$ extra demping

2) steilheid in 3 dB-punt

$$BW : \frac{d\,|H(j\omega)|}{d\omega} = -\frac{1}{2}\cdot\left(1+\omega^{2n}\right)^{-\frac{3}{2}} . 2n . \omega^{2n-1}$$

$$\omega = 1 \Rightarrow -\frac{1}{2} . 2^{-\frac{3}{2}} . 2n = -\frac{n}{2\sqrt{2}}$$ (in 3dB punt)

Tsj $|H(j\omega)| = \dfrac{1}{\sqrt{1+\varepsilon^2 T_n^2(\omega)}}$

$$\frac{d\,|H(j\omega)|}{d\omega} = -\frac{1}{2}\cdot\left(1+\varepsilon^2 T_n^2(\omega)\right)\ 2T_n \cdot T_n'$$

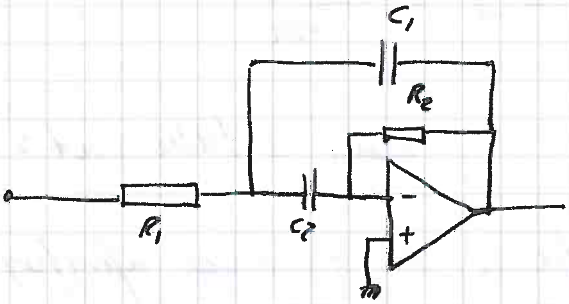
$T_1' = 1$
$T_2' = 4\omega$
$T_3' = 12\omega^2-3$

$T_n = 1$ { $(\omega = 1)$
$T_n' = n^2$

afgel: $-\frac{1}{2}\cdot 2^{-3/2}\cdot 2\,n^2 = -\dfrac{n^2}{2\sqrt{2}}$ in $(\omega=1)$

nu volgt: Banddoorlaat & Bandsper

Opgave:



ga nu dat ►

$$H(s) = \frac{-\frac{1}{R_1C_1}\cdot s}{s^2+\frac{1}{R_2}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)s+\frac{1}{R_1R_2C_1C_2}}$$

Delyiannis - Friend Circuit (DF)

Opgave: Tsj HD

$\alpha_{max} = 1$ dB

$\alpha_{min} = 25$ dB

3000, 6990 rad/sec

$K=1$ C's: 10 nF

Banddoorlaat filter $\quad LD \longleftrightarrow BD$

Eenvoudigste geval :

$$\frac{1}{s} \longrightarrow \frac{s}{(s+jw_0)(s-jw_0)} = \frac{s}{s^2+w_0^2}$$

$$s \longrightarrow \frac{s^2+w_0^2}{sB}$$

$\underline{s=jw}$

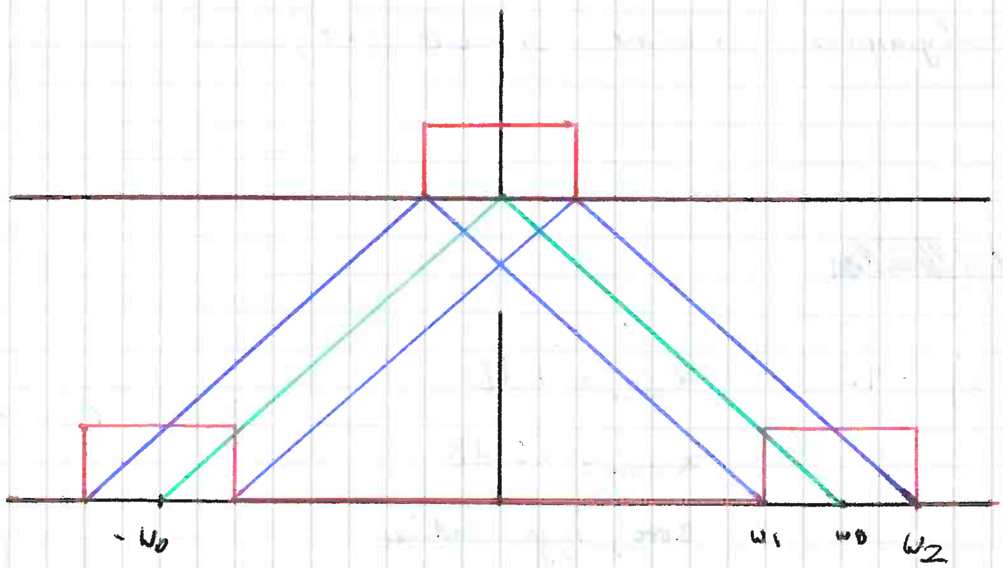
$$jw \rightarrow \frac{-w^2+w_0^2}{jw}$$

$$w \rightarrow \frac{-w_0^2+w^2}{w}$$

wat is het effect van : $\Omega_{Ld} = \dfrac{w_0^2-w_{BD}^2}{w_{BD}}$

$w_{BD} = w_0 \longrightarrow \Omega_{LD} = 0$ $\quad$ neem 1 filter met 2 kar. punten

Bijv. het ideale filter en de twee afsnijfrequenties



We wensen

$$1 = \frac{-w_0^2 + w_2^2}{w_2^2} = -w_0^2 + w_2^2 = w_2$$

(1) $$-1 = \frac{-w_0^2 + w_1^2}{w_1} = -w_0^2 + w_1^2 = -w_1$$

$$\overline{} \; -$$

$$-w_1^2 + w_2^2 = w_2 + w_1$$

$$-w_1 + w_2 = +1$$

Bandbreedte $= 1 \rightarrow$ aanpassen, zie rode wijzigingen

Bandbreedte B:

$$w_2 - w_1 = 1.B$$

$$\Omega_{LD} = \frac{w_{BD}^2 - w_0^2}{w_{BD} \cdot B}$$

Uit (1) volgt dat:

$$w_1^2 - w_0^2 = -w_1 (w_2 - w_1)$$

$$= -w_1 w_2 + w_1^2$$

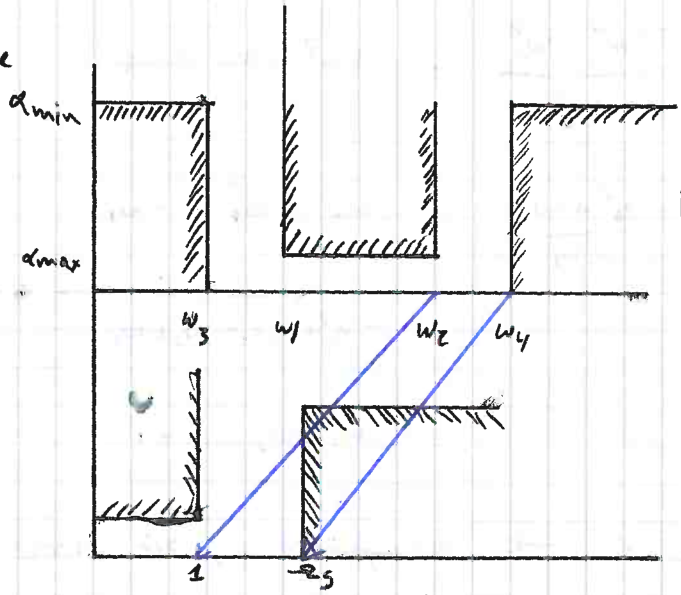
$$\rightarrow \underline{w_0 = \pm \sqrt{w_1 w_2}}$$

Dit geldt voor elk 2-tal karakteristieke punten dus ook bijvoorbeeld bij BW:

$$w_0 = \sqrt{w_{L,10dB} \cdot w_{H,10dB}}$$

Nu het ontwerp

specificatie



$w_0 = \sqrt{w_1 w_2} = \sqrt{w_3 w_4}$

vereist

$w_3 w_4$ te groot $\rightarrow$ w_4 verkleinen

$w_3 w_4$ te klein $\rightarrow$ w_3 vergroten

$$\Omega = \frac{w^2 - w_0^2}{w \cdot B} \qquad \Omega_0 = \frac{1}{\left(10^{\frac{\alpha_{max}}{10}} - 1\right)^{1/2n}}$$

Voor $w = w_2$ krijgen we : $\Omega = 1$ is Ω_p in LD ontwerp

$$W = w_4 : \quad \Omega = \Omega_s = \frac{w_4^2 - w_0^2}{w_4 \cdot B} = \frac{w_4^2 - w_3 w_4}{w_4 (w_2 - w_1)}$$

$$\Omega_s = \frac{w_4 - w_3}{w_2 - w_1}$$

nu laagdoorlaat filter doorrekenen :

stel : $\alpha_{max} = 0,5$ dB $\quad \alpha_{min} = 20$ dB $\quad$ B.W.

$f_3 = 200$ Hz $\quad f_1 = 500$ Hz $\quad f_2 = 1000$ H $\quad f_4 = 2000$ Hz

$n_{LD} = 3 \qquad \Omega_0 = 1,42$ (a_{max} aanhouden)

Ω_{3dB}

$$H(s)_{LD} \quad \frac{1 \qquad 1}{(s+1,42)(s+0,71+j\,1,23)(s+0,71-j\,1,23)}$$

?

Nu terugtransformeren naar B.D.

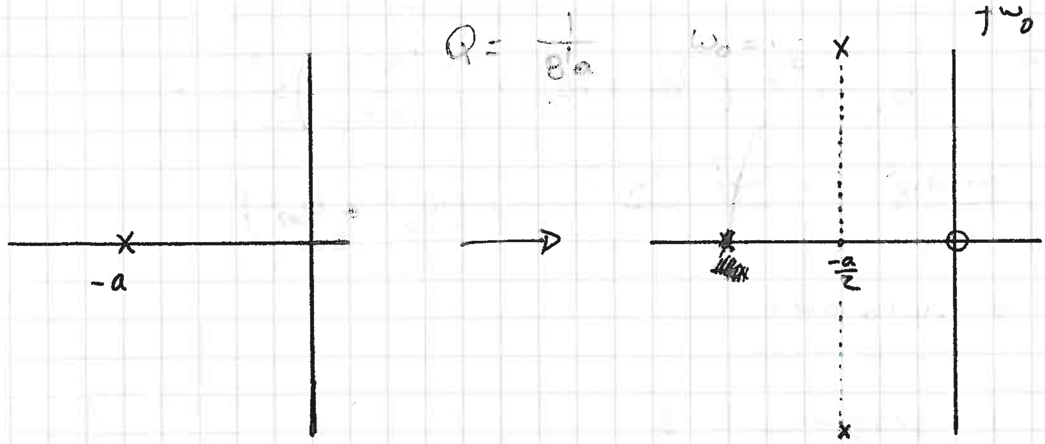
m.bv. $s \longrightarrow \dfrac{s^2+w_0^2}{B.s}$ dit wordt een 6^e orde netwerk.

1^e en 2^e orde-deel apart behandelen

$$\frac{1}{s+a} \rightarrow \frac{1}{\dfrac{s^2+w_0^2}{Bs}+a} = \frac{Bs}{s^2+aBs+w_0^2}$$

polen : $$s = \frac{-aB \pm \sqrt{(aB)^2 - 4w_0^2}}{2}$$

$$= \frac{aB}{2} \pm j\sqrt{w_0^2 - \left(\frac{aB}{2}\right)^2}$$



2^e orde

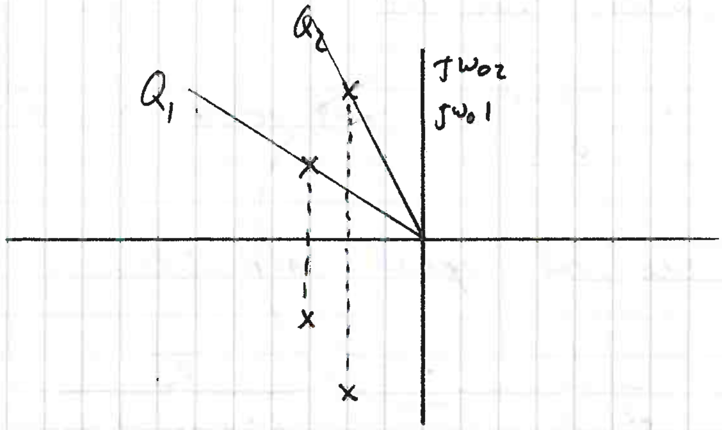
$$\frac{1}{s^2+as+b} \Rightarrow \frac{1}{\left(\dfrac{s^2+w_0^2}{sB}\right)^2 + a\left(\dfrac{s^2+w_0^2}{sB}\right) + b}$$

$$= \frac{B^2s^2}{s^4+2w_0^2s^2+w_0^4+aBs^3+aBw_0^2s+bB^2s^2}$$

$$= \frac{}{s^4+aBs^3+(2w_0^2+bB^2)s^2+aBw_0^2s+w_0^4}$$

Geffe heeft deze 4^e graadsverg. opgelost

Neem aan: cascade van 2 2^e orde circuits met ω_{01} en Q_1 resp. ω_{02} en Q_2



Stel de noemer van deze overdracht gelijk aan

$$\left(s^2 + \frac{\omega_{01}}{Q_1} s + \omega_{01}^2\right)\left(s^2 + \frac{\omega_{02}}{Q_2} s + \omega_{02}^2\right) =$$

$$s^4 + \left(\frac{\omega_{01}}{Q_1} + \frac{\omega_0}{Q_2}\right) s^3 + \left(\omega_{02}^2 + \omega_{01}^2 + \frac{\omega_{01}\omega_{02}}{Q_1 Q_2}\right) s^2 +$$

$$+ \left(\frac{\omega_{01}\omega_{02}^2}{Q_1} + \frac{\omega_{01}^2\omega_{02}}{Q_2}\right) s + \left(\omega_{01}^2 \cdot \omega_{02}^2\right)$$

Stel $\omega_0 = 1$ → normeren ;

d.w.z. B → genormeerde B

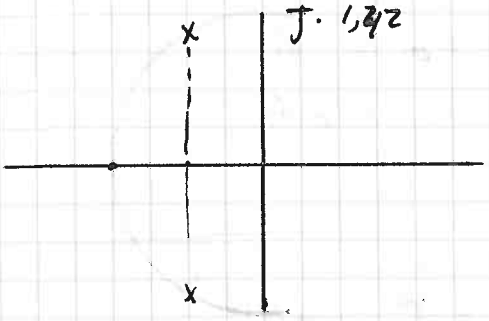
$$\frac{\omega_{01}}{Q_1} + \frac{\omega_{02}}{Q_2} = aB \qquad \omega_{02}^2 + \omega_{01}^2 + \frac{\omega_{01}\omega_{02}}{Q_1 Q_2} = 2 + bB^2$$

→ $\omega_0 = 1$

$$\frac{\omega_{01}\,\omega_{02}^2}{Q_1} + \frac{\omega_{01}^2\,\omega_{02}}{Q_2} = aB$$

$$\omega_{01}^2 \cdot \omega_{02}^2 = 1$$

Doorrekenen van het vb:



polen : $-0{,}71 + J\,1{,}23$

$a = 1{,}42$

$b = 2{,}01$

$$a \cdot B = 1{,}42 \cdot \frac{500}{707} = 1 \qquad \sqrt{w_1 w_2}$$

$$b B^2 = 2{,}01 \cdot (0{,}707)^2 = 1$$

Dan:

1) $$\frac{w_{01}}{Q_1} + \frac{w_{02}}{Q_2} = 1$$

2) $$w_{01}^2 + w_{02}^2 + \frac{w_{01} w_{02}}{Q_1 Q_2} = 3$$

3) $$\frac{w_{01} w_{02}^2}{Q_1} + \frac{w_{01}^2 w_{02}}{Q_2} = 1$$

4) $$w_{01}^2 \cdot w_{02}^2 = 1 \rightarrow w_{01} \cdot w_{02} = 1$$

(5)

(5) $$\frac{w_{02}}{Q_1} + \frac{w_{01}}{Q_2} = 1 \quad (1)$$

verwisselen van Q_1 en Q_2 kan direct

$Q_1 = Q_2 = Q$ $\qquad w_{01} + w_{02} = Q$

uit 2) $$w_{01}^2 + w_{02}^2 + \frac{1}{Q^2} = 3$$

$$(w_{01} + w_{02})^2 - 2 w_{01} w_{02} + \frac{1}{Q^2} = 3$$

$$Q^2 - 2 + \frac{1}{Q^2} = 3$$

$$Q^4 - 5Q^2 + 1 = 0 \qquad Q^2 = 4{,}79 \lor 0{,}21$$

$$Q = 2{,}23$$

dus $w_{01} + w_{02} = 2{,}23$

$w_{01} \cdot w_{02} = 1$

$$w_{01}\ (2{,}23 - w_{01}) = 1$$

$$w_{01}^2 - 2{,}23\ w_{01} + 1 = 0 \qquad w_{01} = 1{,}61$$

$$w_{02} = 0{,}62$$

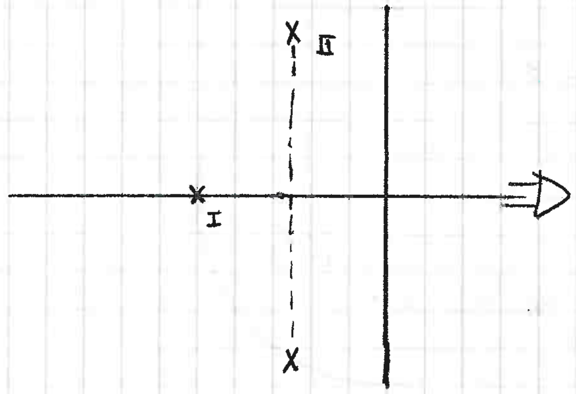
Algemeen :

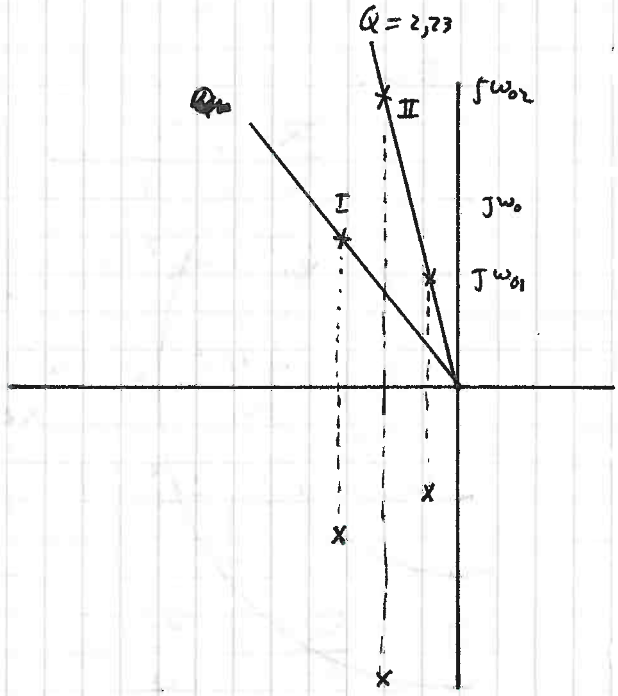
$$Q = \sqrt{\frac{4 + bB^2 + \sqrt{(4+bB^2)^2 - 4(aB)^2}}{2(aB)^2}}$$

B : genormeerd!

$$w_{01,2} = \frac{QaB \pm \sqrt{(QaB)^2 - 4}}{2}$$

Voor het 3e orde BW-filter





~~Deze Uitwerking~~
Deze berekening is vooral interressant voor filters met vrij grote B.
Voor relatief smalbandige filters kan een benadering gehanteerd worden waarbij rond $\pm j w_0$ de polen worden geplot zoals bij LD rond 0. Dit geldt zowel voor BW. als Tsj.

Opgave 1

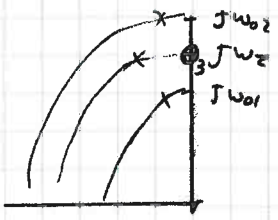
Werk het voorbeeld verder uit

Neem in de DF-circuit $C_1 = C_2 = 0{,}1\mu F$

Opgave 2

Bereken met dezelfde gegevens een TSj. filter

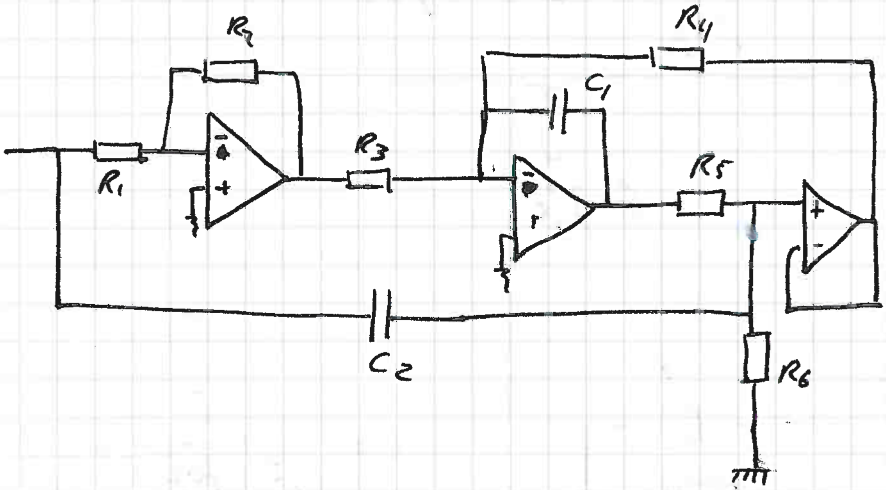
<u>Bandsper filter.</u> (notch-filter)



Voor het ontwerp zijn in het algemeen drie soorten circuits nodig

1. notch
2. LP. notch
3. HP. notch

Alle drie de functies in onderstaand circuit:



neem $\omega_0 = 1$

neem $\frac{R_2}{R_1} = k$

$C_1 = C_2 = 1$

$R_5 = R_6$

Dan:

$$\omega_0^2 = \frac{1}{R_4 R_5}$$

$$\omega_z^2 = \frac{k}{R_3 R_5}$$

$$\frac{\omega_0}{Q} = \frac{R_5 + R_6}{R_5 R_6}$$

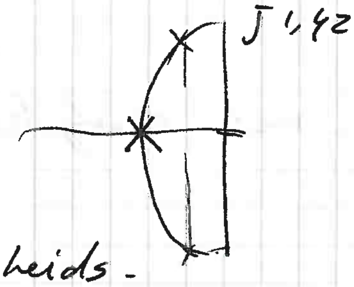
$$Q = \frac{1}{2}\sqrt{\frac{R_5}{R_4}}$$

Ga na:

$$H(s) = \frac{s^2 + \dfrac{R_2}{R_1 R_3 R_5 C_1 C_2}}{s^2 + s \cdot \dfrac{R_5 + R_6}{R_5 R_6 C_2} + \dfrac{1}{R_4 R_5 C_1 C_2}}$$

$$\Omega_3 = \frac{1 \leftarrow \Omega_p}{\left(10^{\frac{0,5}{10}} - 1\right)^{1/6}} \quad \text{of} \quad \Omega_3 = \frac{3,5 \leftarrow \Omega_s}{\left(10^{\frac{20}{10}} - 1\right)^{1/6}}$$

$= 1,42$



1) 1e graadsterm : $(s + 1,42)$

indien gewerkt wordt met eenheids-cirkel, moet genormeerde bandbreedte opnieuw aangepast worden.

$$Q = \frac{1}{B'a} \approx 1$$

2) 2e graadsterm

$$H_2(s) = \frac{1}{(s + 1,42\cos 60 + 1,42\sin 60)\,(s + 1,42\cos 60 - 1,42\sin 60)}$$

$a = 1,42$

$b = 2,017$

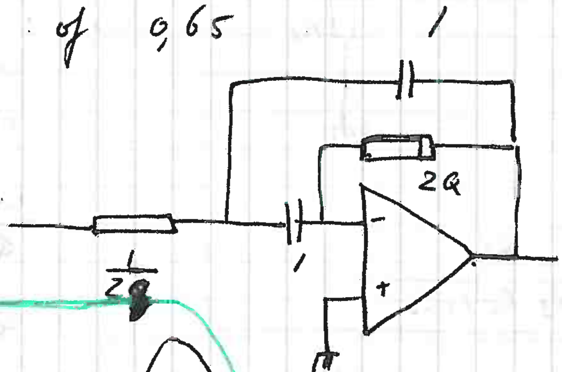
$B' = 0,707$

↓

beffe

$Q = 2,181$

$w_0 = 1,52$ of $0,65$

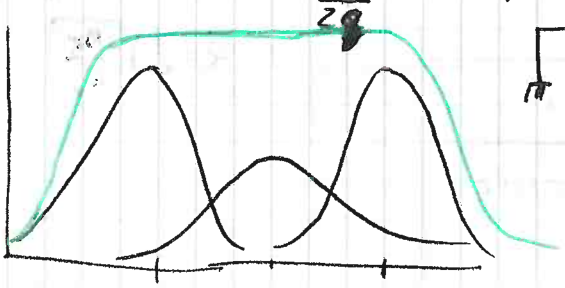


$$C = \frac{1}{1,52 \cdot 707}$$

↑

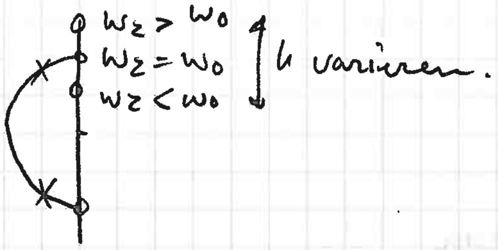
centerfrequentie

$$R_1 = \frac{1}{2 \cdot 2,181}$$



stagger-tuning

Bainter

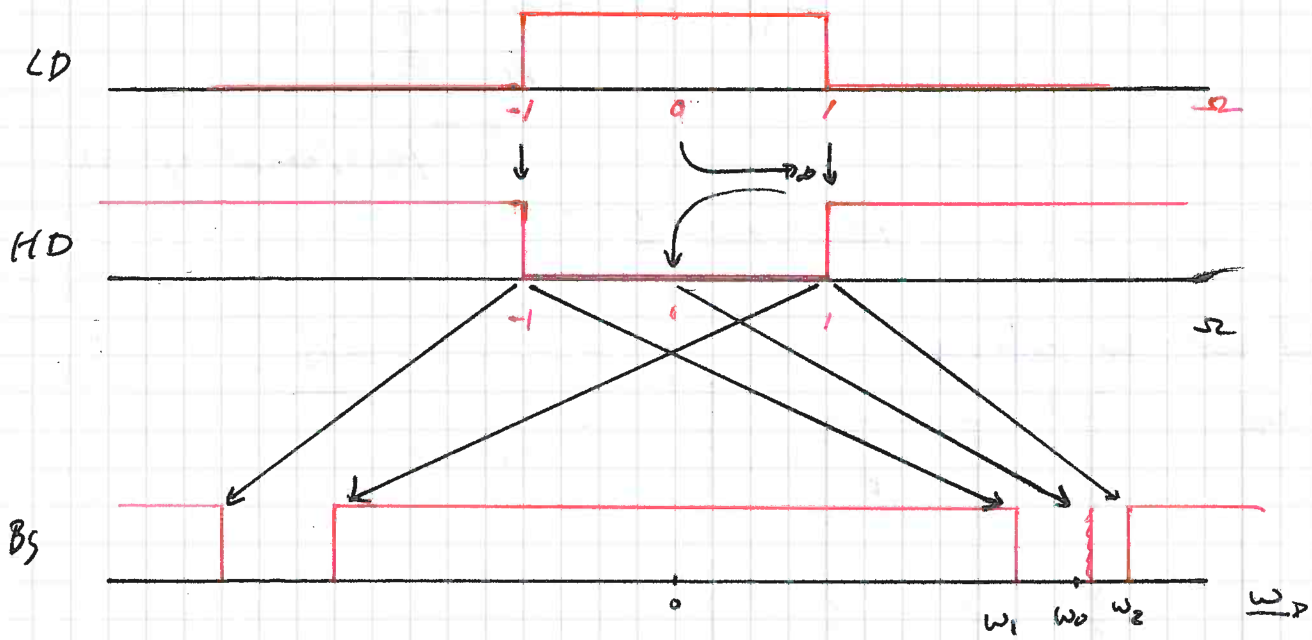


Transformatie Bandsper $\rightleftarrows$ LD

LD $\rightarrow$ HD $\xrightarrow{\text{LD + BD}}$ BS

$$s \rightarrow \frac{1}{s} \longrightarrow \frac{1}{\frac{s^2+\omega_0^2}{Bs}} = \frac{Bs}{s^2+\omega_0^2}$$

$$j\omega \rightarrow \frac{Bj\omega}{\omega_0^2-\omega^2} \qquad \omega \rightarrow \frac{B\omega}{\omega_0^2-\omega^2}$$



en mit

$$\left.\begin{aligned} 1 &= \frac{B\omega_2}{\omega_0^2-\omega_2^2} \\ -1 &= \frac{B\omega_1}{\omega_0^2-\omega_1^2} \end{aligned}\right\} \quad (B = \omega_2 - \omega_1)$$

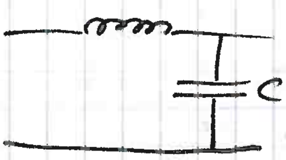
volgen nu de omrekeningsregels

Ontwerpvolgorde :

1) BS specs naar LD
2) LD equivalent ontwerpen
3) Overgang naar HD : spiegelen in e.s.
4) overgang naar BS (geffe)

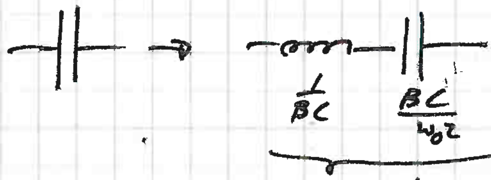
Illustratie m.b.v. ladder filter

LD:



$$C := \frac{1}{sC} \longrightarrow \frac{1}{\frac{Bs}{s^2+\omega_0^2} \cdot C} = \frac{s^2+\omega_0^2}{BsC}$$

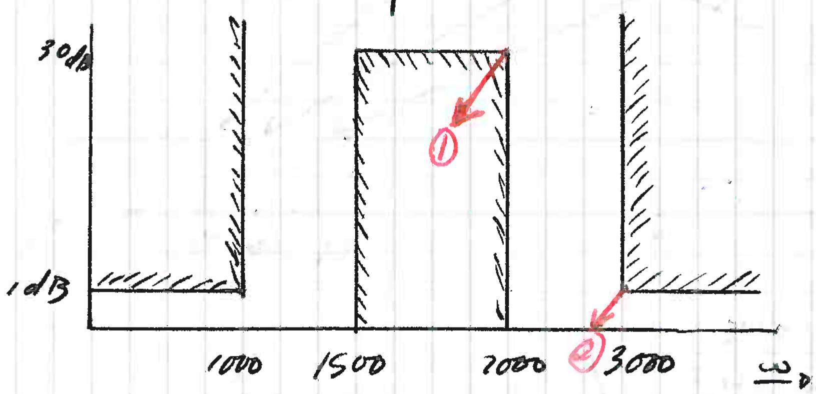
$$= \frac{s}{BC} + \frac{1}{s\frac{BC}{\omega_0^2}}$$



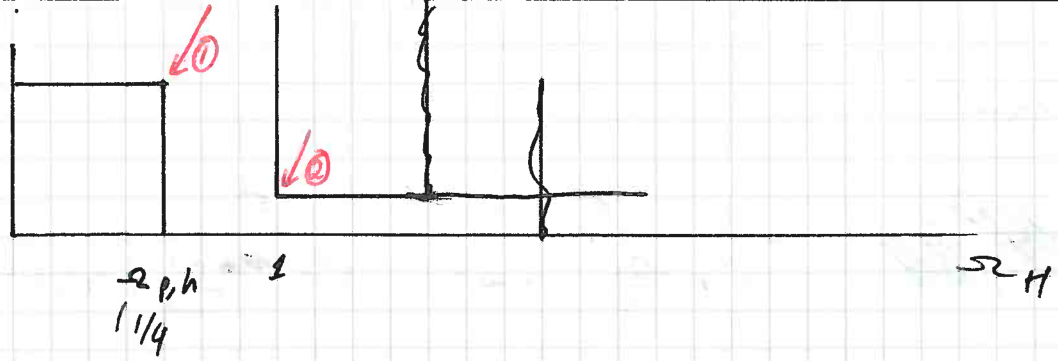
idem voor L

Voorbeeld van een ontwerp:

BW :

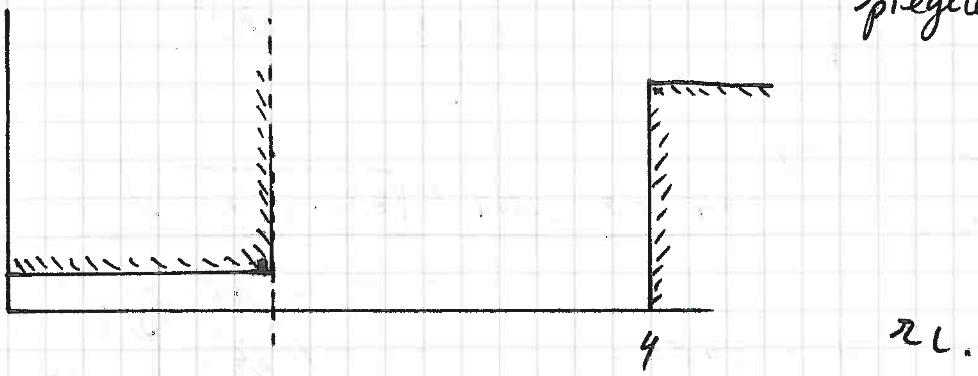


$$\omega_0 = \sqrt{1500 \cdot 2000}$$
$$\sqrt{1000 \cdot 3000} = 1732 \text{ rad/s}.$$



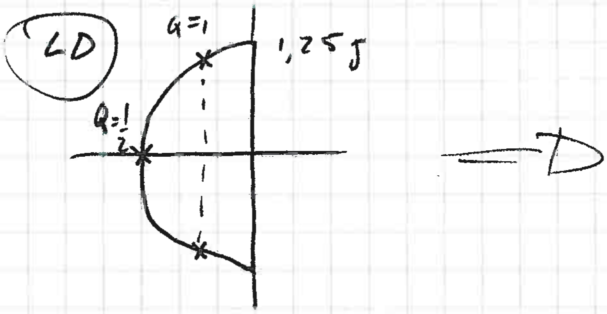
$$\Omega_{p,h} = \frac{2000-1500}{3000-1000} = \frac{1}{4} = \frac{\omega_2-\omega_1}{\omega_4-\omega_3}$$

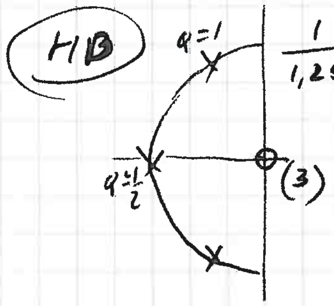
Spiegelen in eenheidscirkel



$$n = \frac{\log\left[(10^{3}-1)/(10^{0,1}-1)\right]}{2\log 4} = 2,978 \longrightarrow 3$$

$$\Omega_0 = \frac{1}{(10^{0,1}-1)^{1/6}} = 1,25$$





$\frac{1}{1,25}$ j $= 0,8$ j

(spiegelen in de eenheidscirkel)

nu Beffe:

1e graadsterm $\frac{s}{s+0,8}$

$B' = \frac{2000}{1732} = 1,1547$

$Q = \frac{1}{B' \cdot a} = \frac{1}{1,1547 \cdot 0,8} = \underline{\underline{1,082}}$

$\omega_0 = 1$

2^e graadsterm

$$\frac{s^2}{s^2 + \frac{\omega_0}{Q}\cdot s + \omega_0^2} \qquad \frac{s^2}{s^2 + \frac{0,8}{1,08}\, s + (0,8)^2}$$

Geffe :

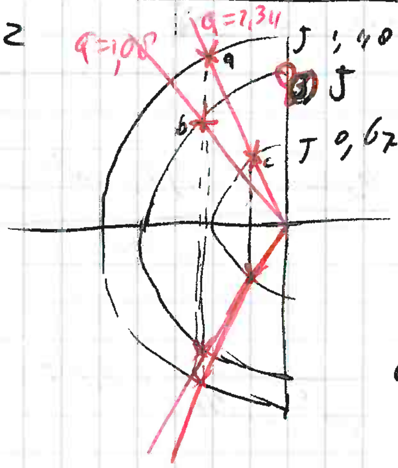
$$Q = \sqrt{\frac{4 + 0,64 \cdot 1,1547^2 + \sqrt{(4 + 0,64 \cdot 1,1547^2)^2 - (4 \cdot 0,8 \cdot 1,1547^2)}}{2(0,8 \cdot 1,1547)^2}}$$

$Q = 2,34$

$$\omega_{01,02} = \frac{2,34 \cdot 0,8 \cdot 1,1547 \pm \sqrt{(2,34 \cdot 0,8 \cdot 1,1547)^2 - 4}}{2}$$

$\omega_{01} = 1,48$

$\omega_{02} = 0,67$



a Hp-notch
b notch
c lp-notch.

Bainter

$\omega_z^2 = k$ (nulpunten)

$\omega_0 = 1$ (polen)

$R_5 = R_6 = 2Q$

$R_4 = \frac{1}{2Q} = R_3$

lp-notch $= R_3 = R_4 = \frac{1}{2 \cdot 2,34} \cdot k_i$ $\qquad C = \frac{1}{0,67 \cdot 1732} \cdot k_i$

$R_5 = R_6 = 2 \cdot 2,34 \cdot k_i$

$k = \omega_z^2 = \left(\frac{1}{0,67}\right)^2$